Amending Maxwell's Equations for Real and Complex Gauge Groups in Non-Abelian Form

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Abstract. We have analyzed, calculated and extended the modification of Maxwell's equations in a complex Minkowski metric, M_4 in a C_2 space using the SU_2 gauge, SL(2,c) and other gauge groups, such as SU_n for $n > 2$ expanding the U_1 gauge theories of Weyl. This work yields additional predictions beyond the electroweak unification scheme. Some of these are: 1) modified gauge invariant conditions, 2) short range non-Abelian force terms and Abelian long range force terms in Maxwell's equations, 3) finite but small rest of the photon, and 4) a magnetic monopole like term and 5) longitudinal as well as transverse magnetic and electromagnetic field components in a complex Minkowski metric M_4 in a C_4 space.

INTRODUCTION

We have developed an eight dimensional complex Minkowski space $M_4(1)$ composed of four real dimensions and four imaginary dimensions which is consistent with Lorentz invariance and analytic continuation in the complex plane [1]. The unique feature of this geometry is that it admits of nonlocality consistent with Bell's theorem, (EPR paradox), possibly Young's double slit experiment, the Aharonov – Bohm effect and multi mirrored interferometric experiment.

Additionally, expressing Maxwell's electromagnetic equations in complex eight space, leads to some new and interesting predictions in physics, including possible detailed explanation of some of the previously mentioned nonlocality experiments [2]. Complexification of Maxwell's equations require a non-Abelian gauge group which amend the usual theory, which utilizes the usual unimodular Weyl U_1 group. We have examined the modification of gauge conditions using higher symmetry groups such as SU_2, SU_n and other groups such as the SL(2,c) double cover group of the rotational group SO(3,1) related to Shipov's Ricci curvature tensor [3] and a possible neo-aether picture. Thus we are led to new and interesting physics involving extended metrical space constraints, the usual transverse and also longitudinal, non Hertzian electric and magnetic field solutions to Maxwell's equations, possibly leading to new communication systems and antennae theory, non zero solutions to $\nabla \cdot \underline{B}$, and a possible finite but small rest mass of the photon.

Comparison of our theoretical approach is made to the work of Vigier, [4] Barrett [5] and Harmuth's [6] work on amended Maxwell's theory. We compare our predictions such as our longitudinal field to the $B^{(3)}$

term of Vigier, and our Non-Abelian gauge groups to that of Barrett and Harmuth. This author interprets this work as leading to new and interesting physics, including a possible reinterpretation of a neo-aether with nonlocal information transmission properties.

COMPLEXIFIED ELECTROMAGNETIC FIELDS IN MINKOWSKI SPACE AND NONLOCALITY

We expand the usual line element metric $dg^2 = g\mu\gamma dx^{\nu} dx$ in the following manner. We consider a complex eight dimensional space, M_4 constructed so that $Z^u = \chi^u_{\mathrm{Re}} + i\,\chi^u_{\mathrm{Im}}$ and likewise for Z^{ν} where the indices ν and μ run 1 to 4 yielding (1, 1, 1, -1). Hence, we now have a new complex eight space metric as $ds^2 = \eta_{\nu\mu} dZ^{\nu} dZ^{\mu}$. We have developed this space and other extended complex spaces (1) and examined their relationship with the twister algebras and asymptotic twister space and the spinor calculus and other implications of the theory [7]. The Penrose twister SU(2,2) or U_4 is constructed from 4-space – time, $U_2 \otimes \widetilde{U}_2$ where U_2 is the real part of the space and $\widetilde{U}_2$ is the imaginary part of the space, this metric appears to be a fruitful area to explore.

The twister *Z* can be a pair of spinors U^A and π_A which are said to represent the twister. The condition for these representations are 1) the null infinity condition for a zero spin field is $Z^e \overline{Z}_e = 0$, 2) conformal invariance and 3) independence of the origin. The twister is derived from the imaginary part of the spinor field. The underlying concept of twister theory is that of conformally invariance fields occupy a fundamental role in physics and may yield some new physics. Since the twister algebra falls naturally out complex space.

Other researchers have examined complex dimensional Minkowski spaces. In reference [10], Newman demonstrates that M_4 space do not generate any major “weird physics” or anomalous physics predictions and is consistent with an expanded or amended special and general relativity. In fact the Kerr metric falls naturally out of this formalism as demonstrated by Newman [11].

As we know twisters and spinors are related by the general Lorentz conditions in such a manner that all signals are luminal in the usual four N Minkowski space but this does not preclude super or trans luminal signals in spaces where N > 4. H. Stapp, for example, has interpreted the Bell’s theorem experimental results in terms of trans luminal signals to address the nonlocality issue of the Clauser, et. al and Aspect experiments. Kozameh and Newman demonstrate the role of non local fields in complex eight space [16].

We believe that there are some very interesting properties of the M_4 space which include the nonlocality properties of the metric applicable in the non-Abiliass algebras related to the quantum theory and the conformal invariance in relativity as well as new properties of Maxwell’s equations. In addition, complexification of Maxwell’s equations in M_4 space yields some interesting predictions, yet we find the usual conditions on the manifold hold [2,8]. Some of these new predictions come out of the complexification of four space 2 and appear to relate to the work of Vigier, Barrett, Harmuth and others [4, 5, 6]. Also we find that the twister algebra of the complex eight dimensional, M_4 space is mapable 1 to 1 with the twister algebra, C_4 space of the Kaluza-Klein five dimensional electromagnetic - gravitational metric [17, 18].

Some of the predictions of the complexified form of Maxwell’s equations are 1) a finite but small rest mass of the photon, 2) a possible magnetic monopole, $\nabla \cdot \beta \neq 0$, 3) transverse as well as longitudinal B(3) like components of $\underline{E}$ and $\underline{B}$, 4) new extended gauge invariance conditions to include non-Abelian algebras and 5) an inherent fundamental nonlocality property on the manifold. Vigier also explores longitudinal $\underline{E}$ and $\underline{B}$ components in detail and finite rest mass of the photon [19].

We consider both the electric and magnetic fields to be complexified as $\underline{E} = E_{\mathrm{Re}} = iE_{\mathrm{Im}}$ and $\underline{B} - \underline{B}_{\mathrm{Re}} = iB_{\mathrm{Im}}$ for $E_{\mathrm{Re}}, E_{\mathrm{Im}}, B_{\mathrm{Re}}$ and B_{Im} are real quantities. Then substitution of these two equations into the complex form of Maxwell's equations above yields, upon separation of real and imaginary parts, two sets of Maxwell-like equations. The first set is

$$\nabla \cdot \underline{E}_{\mathrm{Re}} = 4\pi \rho_e \tag{1}$$

$$\nabla \cdot \underline{B}_{\mathrm{Re}} = 0 \qquad \nabla \times \underline{B}_{\mathrm{Re}} = \frac{1}{c}\frac{\partial \underline{E}_{\mathrm{Re}}}{\partial t} = \underline{J}_e \tag{2}$$

the second set is

$$\nabla \cdot \left(i\underline{B}_{\mathrm{Im}}\right) = 4\pi i \rho_m \qquad \nabla \times \left(i\underline{B}_{\mathrm{Im}}\right) = \frac{1}{c}\frac{\partial\left(i\underline{E}_{\mathrm{Im}}\right)}{\partial t} \tag{3}$$

$$\nabla \cdot \left(iE_{\mathrm{Im}}\right) = 0 \qquad \nabla \times \left(i\underline{E}\right) = \frac{1}{c}\frac{\partial\left(iB_{\mathrm{Im}}\right)}{\partial t} = i\underline{J}m \tag{4}$$

The real part of the electric and magnetic fields yield the usual Maxwell's equations and complex parts generate "mirror" equations; for example, the divergence of the real component of the magnetic field is zero, but the divergence of the imaginary part of the electric field is zero, and so forth. The structure of the real and imaginary parts of the fields is parallel with the electric real components being substituted by the imaginary part of the magnetic fields and the real part of the magnetic field being substituted by the imaginary part of the electric field.
In the second set of equations, (2), the *I*'s, "go out" so that the quantities in the equations are real, hence $\nabla \cdot B_{\mathrm{Im}} = 4\pi\rho_m$, and not zero, yielding a term that may be associated with some classes of monopole theories. See references in ref. [2].

We express the charge density and current density as complex quantities based on the separation of Maxwell's equations above. Then, in generalized form $\rho = \rho_e = i\rho_m$ and $J=J_e+iJ_m$ where it may be possible to associate the imaginary complex charge with the magnetic monopole and conversely the electric current has an associated imaginary magnetic current.

The alternate of defining and using, which Evans does $\underline{E} = \underline{E}_{\mathrm{Re}} + i\underline{B}_{\mathrm{Im}}$ and $\underline{B} = \underline{B}_{\mathrm{Re}} + i\underline{E}_{\mathrm{Im}}$ would not yield a description of the magnetic monopole in terms of complex quantities but would yield, for example $\nabla \cdot \left(iB_{\mathrm{Im}}\right) = 0$ in the second set of equations.

Using the invariance of the line element $s^2 = x^2 - c^2t^2$ for $r = ct = \sqrt{x}^{\,2}$ and for $x^2 = x^2 + y^2 + z^2$ for the distance from an electron charge, we can write the relation,

$$\frac{1}{c}\frac{\partial\left(iB_{im}\right)}{\partial t} = iJm \qquad \text{or} \qquad \frac{1}{c}\frac{\partial B_{im}}{\partial t} = J_m \tag{5}$$

$$\nabla \times \left(iE_{\mathrm{Im}} \right) = 0 \text{ for } \underline{E}_{\mathrm{Im}} = 0 \qquad \text{or} \qquad \frac{1}{c}\frac{\partial\left(iB_{\mathrm{Im}} \right)}{\partial t} = iJm \tag{6}$$

IMPLICATIONS FOR PHYSICS OF NEW GAUGE CONDITIONS IN COMPLEX MINKOWSKI SPACE

In a series of papers, Barrett, Harmuth and Rauscher have examined the modification of gauge conditions in modified or amended Maxwell theory. The Rauscher approach, as briefly explained in the preceding section is to write complexified Maxwell's equation in consistent form to complex Minkowski space [2].

The Barrett amended Maxwell theory utilizes non-Abelian algebras and leads to some very interesting predictions which have interested me for some years. He utilizes the non commutive SU_2 gauge symmetry rather than the U_1 symmetry. Although the Glashow electroweak theory utilizes U_1 and SU_2, but in a different manner, but his theory does not lead to the interesting and unique predictions of the Barrett theory. Barrett, in his amended Maxwell theory, predicts that the velocity of the propagation of signals is not the velocity of light. He presents the magnetic monopole concept resulting from the amended Maxwell picture. His motive goes beyond standard Maxwell formalism and generate new physics utilizing a non-Abelian gauge theory.[5]

The SU_2 group gives us symmetry breaking to the U_1 group which can act to create a mass splitting symmetry that yield a photon of finite (but necessarily small) rest mass which may be created as self energy produced by the existence of the vacuum. This finite rest mass photon can constitute a propagation signal carrier less than the velocity of light.

We can construct the generators of the SU2 algebra in terms of the fields $\underline{E}$, $\underline{B}$, and $\underline{A}$. The usual potentials, A_μ is the important four vector quality $A_\mu = \left(\underline{A}, \phi \right)$ where the index runs 1 to 4. One of the major purposes of introducing the vector and scalar potentials and also to subscribe to their physicality is the desire by physicists to avoid action at a distance. In fact in gauge theories A_μ is all there is! Yet, it appears that, in fact, these potentials yield a basis for a fundamental nonlocality!

Let us address the specific case of the SU_2 group and consider the elements of a non-Abelian algebra such as the fields with SU_2 (or even SU_n) symmetry then we have the commutation relations where XY-YX $\neq 0$ or $[X,Y] \neq 0$. Which is reminiscent of the Heisenberg uncertainty principle non-Abelian gauge. Barrett does explain that SU_2 fields can be transformed into U_1 fields by symmetry breaking. For the SU_2 gauge amended Maxwell theory additional terms appear in term of operations such $A \cdot E, A \cdot B$ and $A \times B$ and their non Abelian converses. For example $\nabla \cdot B$ no longer equals zero but is given as $\nabla \cdot B = -jg\left(A \cdot B - B \cdot A \right) \neq 0$ where $[A,B] \neq 0$ for the dot product of A and B and hence we have a magnetic monopole term and j is the current and g is a constant. Also Barrett gives references to the Dirac, Schwinger and G. t Hooft monopole work. Further commentary on the SU_2 gauge conjecture of H.F. Mamuth [6] that under symmetry breaking, electric charge is considered but magnetic charges are not. Barrett further states that the symmetry breaking conditions chosen are to be determined by the physics of the problem. These non Abelian algebras have consistence to quantum theory.

In this author's analysis, using the SU_2 group there is the automatic introduction of short range forces in addition to the long range force of the U_1 group. U_1 is one dimensional and Abelian and SU_2 is three dimensional and is non-Abelian. U_1 is also a subgroup of SU_2. The U_1 group is associated with the long range $1/r^2$ force and SU_2, such as for its application to the weak force yields short range associated fields. Also SU_2 is a subgroup of the useful SL(2,c) group of non compact operations on the manifold. SL(2,c) is a semi simple four dimensional Lie group and is a spinor group relevant to the relativistic formalism and is isomorphic to the connected Lorentz group associated with the Lorentz transformations. It is a conjugate group to the SU_2 group and contains an inverse. The double cover group of SU_2 is SL(2,c) where SL(2,c) is a complexification of SU_2. Also LS(2,c) is the double cover group of SU_3 related to the set of rotations in three dimensional space [3]. Topologically, SU_2 is associated with isomorphic to the three dimensional spherical, O_3^+ (or three dimensional rotations) and U_1 is associated with the O_2 group of rotations in two dimensions. The ratio of Abelian to non Abelian components, moving from U_1 to SU_2, gauge is 1 to 2 so that the short range components are twice as many as the long range components.

Instead of using the SU_2 gauge condition we use SL (2,c) we have a non-Abelian gauge and hence quantum theory and since this group is a spinor and is the double cover group of the Lorentz group (for spin ½) we have the conditions for a relativistic formalism. The Barrett formalism is non-relativistic. SL (2,c) is the double cover group of SU_2 but utilizing a similar approach using twister algebras yields relativistic physics.

It appears that complex geometry can yield a new complementary unification of quantum theory, relativity and allow a domain of action for nonlocality phenomena, such as displayed in the results of the Bell's theorem tests of the EPR paradox [22], and in which the principles of the quantum theory hold to be universally. The properties of the nonlocal connections in complex four space may be mediated by non -or low dispersive loss solutions. We solved Schrödinger equation in complex Minkowski space [25].

In progress is research involving other extended gauge theory models, with particular interest in the nonlocality properties on the S pact-time manifold, quantum properties such as expressed in the EPR paradox and coherent states in matter.

Utilizing Coxeter graphs or Dynkin diagrams, Sirag lays out a comprehensive program in terms of the A_n, D_n and E_6, E_7 and E_8 Lie algebras constructing a hyper dimensional geometry for as a classification scheme for elementary particles. Inherently, this theory utilizes complexified spaces involving twisters and Kaluza-Klein geometries. This space incorporates the string theory and GUT models [27].

CONCLUSION

It appears that utilizing the complexification of Maxwell's equations with the extension of the gauge condition to non-Abelian algebras, yields a possible metrical unification of relativity, electromagnetism and quantum theory. This unique new approach yields a universal nonlocality. No radical spurious predictions result from the theory, but some new predictions are made which can be experimentally examined. Also, this unique approach in terms of the twister algebras may lead to a broader understanding of macro and micro nonlocality and possible transverse electromagnetic fields observed as nonlocality in collective plasma state and other media.

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