Combined Gravitational Action-Third Part: Exploration[1]

Mohamed E. Hassani
Institute for Fundamental Research
BP.197, CTR, GHARDAIA 47800, ALGERIA
E-mail: hassani641@gmail.com

Abstract: In previous papers [1,2] relating to the concept of Combined Gravitational Action (CGA) we have established the CGA-theoretical foundations as an alternative gravity theory that already allowed us to resolve -in its context- some unexpected and defiant problems occurred inside and outside the Solar System like, *e.g.*, the anomalous Pioneer 10's deceleration; the observed secular increase of the Astronomical Unit [3] and the apsidal motion anomaly of the eclipsing binary star systems AS Camelopardalis and DI Herculis. All that has been done without exploring fully the CGA-formalism, hence, the main purpose of the present paper is to explore and exploit profoundly the CGA-equations in order to investigate, among other things, the secular perigee precession of the Moon; the secular perihelion advance of the planets; the CGA-effects in the non-compact and compact stellar objects.

Keywords: combined gravitational action; combined gravitational potential energy; Newton's law of gravitation; solar system; eclipsing binary stars; binary pulsars

1. Introduction

Basing only on Euclidean geometry and Galilean relativity principle, we were able to formulate a coherent alternative gravity theory exclusively founded on the concept of Combined Gravitational Action. We have previously [1,2] shown that the theory (CGA) is very capable of predicting and explaining the anomalous Pioneer 10's deceleration; the secular perihelion precession of the inner planets and the angular deflection of light passing near the massive object. These two last phenomena are known as the crucial tests support the general relativity theory (GRT). Here, our main motivation is the following: since in the previous papers [1,2] we did not explore and exploit fully the CGA-formalism, hence, now it is time to do this in order to study, among other things, the CGA-effects in the non-compact stellar objects like, *e.g.*, the eclipsing binary star systems and the compact stellar objects like ,*e.g.*, the binary neutron stars and pulsars.

Before the advent of the CGA as an alternative gravity theory, it was always stressed that the study of such compact stellar objects is exclusively belonging to GR-domain because their strong compactness is enough to bend the local space-time in such a way that some observable GR-effects should occur. However, as we shall see, the CGA is also able to investigate, predict and explain the same type of the compact stellar objects and all that in the context of Euclidean geometry and Galilean relativity principle. This reflects a tangible fact that the propagation of gravitational field and the action of gravitational force both are independent of the *topology* of space-time. But why shall the CGA arrive at the same results as GRT or even better in some cases? Because if we take the concept of the curvature of space-time apart, we find that contrary to the Newton's gravity theory, the CGA and GRT take, at the same time, in full consideration the relative motion of the test-body and the light speed in local vacuum which in CGA is playing the role of a specific kinematical parameter of normalization and in GRT is considered as the speed of gravity propagation. The main consequence of the CGA-formalism [1,2] is the dynamic gravitational field (DGF), $\mathbf{\Lambda}$, which is in reality an induced field, it is more precisely a sort of gravitational induction due to the relative motion of material body in the vicinity of the gravitational source. Certainly, the static gravitational field

$$\boldsymbol{\gamma} = -\nabla \phi(r), \tag{1}$$

is in general always stronger than DGF but $\mathbf{\Lambda}$ has its proper role and effects. For example, as an additional field, $\mathbf{\Lambda}$ is responsible for the perihelion advance of Mercury and other planets of the System

[1] This paper is dedicated to the memory of Prof. José Leite Lopes, 28 October 1918 – 12 June 2006.

Solar as we have already seen in [1,2]. Curiously, in his 1912 argument, Einstein himself noted that the inertia of energy and the equality of inertial and gravitational mass lead us to expect that *"gravitation acts more strongly on a moving body than on the same body in case it is at rest."* It seems Einstein's remark reflects very well the expression of the combined gravitational field [1]:

$$\mathbf{g} = \boldsymbol{\gamma} + \boldsymbol{\Lambda}\,. \tag{2}$$

It is clear from (2), that the combined gravitational field, $\mathbf{g}$, may be reduced to the static gravitational field, $\boldsymbol{\gamma}$, only for the case $\boldsymbol{\Lambda} = 0$, that is, when the material test-body under the action of field is at the relative rest with respect to the main gravitational source. Furthermore, as we know from the first paper [2], the combined gravitational field is derived from the combined gravitational potential energy (CGPE) which, here, is *velocity-dependent-CGPE* defined by the expression

$$U \equiv U(r,v) = -\frac{k}{r}\left(1 + \frac{v^2}{w^2}\right), \tag{3}$$

where $k = GMm$; G being the Newton's gravitational constant; M and m are the masses of the gravitational source A and the moving test-body B; $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ is the relative distance between A and B; $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ is the velocity of the test-body B relative to the inertial reference frame of source A; and w is a specific kinematical parameter having the dimensions of a constant velocity defined by

$$w = \begin{cases} c_0, \text{ if } B \text{ is in relative motion inside the vicinity of } A \\ v_{\text{esc}} = \sqrt{2GM/R}, \text{ if } B \text{ is in relative motion outside the vicinity of } A \end{cases}, \tag{4}$$

where c_0 is the light speed in local vacuum and v_{esc} is the escape velocity at the surface of the gravitational source *A*. It is worthwhile to note that the expression (3) shows us more explicitly that the. Moreover, the CGPE (3) constitutes a fundamental solution to a system of three second order partial differential equations, called *'potential equations'* because U is a common solution to these three equations. Indeed, it is easy to show under some appropriate boundary conditions that the combined potential field U is really a fundamental solution to the following equations:

$$\frac{\partial^2 U}{\partial r^2} + \frac{2}{r}\frac{\partial U}{\partial r} = 0\,, \tag{5}$$

$$\frac{\partial^2 U}{\partial v^2} - \frac{1}{v}\frac{\partial U}{\partial v} = 0\,, \tag{6}$$

$$\frac{\partial^2 U}{\partial r\,\partial v} + \frac{1}{r}\frac{\partial U}{\partial v} = 0\,. \tag{7}$$

Since Eqs.(5-7) are homogeneous and admit the same potential function U as a fundamental solution this implies, among other things, that the test-body B is in state of motion at the relative velocity, v, sufficiently far from the main gravitational source *A*. also, as we shall see, the same fundamental solution is the *origin* of the CGA-equations of motion and the CGA-field equations because, as we have previously seen in the second paper [2], the potential function U is a basic part of the CGA-*Lagrangian*.

2. CGA-Equations of motion

Now, we are arrived at the first part of our main subject, that is to say, the exploration and exploitation of the CGA as an alternative gravity theory. Thus, we shall show the relationship between CGA-equations of motion and those of Newton. For a moving test-body *B* of mass *m* characterized by the CGA-*Lagrangian* [2] and evolving under the action of the combined gravitational field $\mathbf{g}$, there is a system of partial differential equations of motion derived from the CGA-*Lagrangian* like so:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}}\right) - \dfrac{\partial L}{\partial x} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{y}}\right) - \dfrac{\partial L}{\partial y} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{z}}\right) - \dfrac{\partial L}{\partial z} = 0 \end{cases}, \tag{8}$$

Where $L = T - U$ is the CGA-*Lagrangian*; $T = \frac{1}{2} m v^2$ and $U \equiv U(r, v)$ are, respectively, the kinetic energy and the combined gravitational potential energy that characterized the test-body *B*. With $\dot{x} = v_x$, $\dot{y} = v_y$, $\dot{z} = v_z$, $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ and $r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$. After performing some differential and algebraic calculations, we get the analytical expressions of the expected CGA-equations of motion

$$\begin{cases} \left(1 + \dfrac{2GM}{w^2 r}\right)\dfrac{dv_x}{dt} + \left(1 + \dfrac{v^2}{w^2}\right)\dfrac{GM}{r^2}\dfrac{x - x_0}{r} = 0 \\ \left(1 + \dfrac{2GM}{w^2 r}\right)\dfrac{dv_y}{dt} + \left(1 + \dfrac{v^2}{w^2}\right)\dfrac{GM}{r^2}\dfrac{y - y_0}{r} = 0 \\ \left(1 + \dfrac{2GM}{w^2 r}\right)\dfrac{dv_z}{dt} + \left(1 + \dfrac{v^2}{w^2}\right)\dfrac{GM}{r^2}\dfrac{z - z_0}{r} = 0 \end{cases}. \tag{9}$$

Or in compact form, we have

$$\frac{d\mathbf{v}}{dt} + \left(1 + \frac{v^2}{w^2}\right)\left(1 + \frac{2GM}{w^2 r}\right)^{-1} \nabla\phi = 0\,, \tag{10}$$

where

$$\phi \equiv \phi(r) = -\frac{GM}{r}\,, \tag{11}$$

is the static gravitational potential. Further, it is clear from (10), when $(2GM / w^2 r) \ll 1$ and $(v / w)^2 \ll 1$, Eq.(10) reduces to the well-known classical equation of motion

$$\frac{d\mathbf{v}}{dt} + \nabla\phi = 0\,. \tag{12}$$

3. CGA-Field Equations

Since during its motion, the test-body is characterized by the combined gravitational potential energy and evolving under the action of the combined gravitational field $\mathbf{g}$, therefore, the CGA-field equations derived from the potential function $U \equiv U(r, v)$ are:

$$\mathbf{g}:\begin{cases} g_x = m^{-1}\left[\dfrac{d}{dt}\left(\dfrac{\partial U}{\partial \dot{x}}\right) - \dfrac{\partial U}{\partial x}\right] \\ g_y = m^{-1}\left[\dfrac{d}{dt}\left(\dfrac{\partial U}{\partial \dot{y}}\right) - \dfrac{\partial U}{\partial y}\right] \\ g_z = m^{-1}\left[\dfrac{d}{dt}\left(\dfrac{\partial U}{\partial \dot{z}}\right) - \dfrac{\partial U}{\partial z}\right] \end{cases}. \tag{13}$$

After performing some differential and algebraic calculations, we obtain the analytical expressions of the expected CGA-field equations

$$\mathbf{g}:\begin{cases} g_x = -\dfrac{2GM}{w^2 r}\dfrac{dv_x}{dt} - \left(1+\dfrac{v^2}{w^2}\right)\dfrac{GM}{r^2}\dfrac{x-x_0}{r} \\ g_y = -\dfrac{2GM}{w^2 r}\dfrac{dv_y}{dt} - \left(1+\dfrac{v^2}{w^2}\right)\dfrac{GM}{r^2}\dfrac{y-y_0}{r} \\ g_z = -\dfrac{2GM}{w^2 r}\dfrac{dv_z}{dt} - \left(1+\dfrac{v^2}{w^2}\right)\dfrac{GM}{r^2}\dfrac{z-z_0}{r} \end{cases}. \tag{14}$$

Or in compact form, we have

$$\mathbf{g} = -\left(1+\frac{v^2}{w^2}\right)\nabla\phi - \frac{2GM}{w^2 r}\frac{d\mathbf{v}}{dt}. \tag{15}$$

Noting that the last quantity, on the right hand side of Eq.(15) is the rate change of new physical quantity called in the context of CGA *'gravitational momentum'* as we will see. Moreover, let us now deduce the classical field equation. To this aim, it is best to note that for the case $(v/w)^2 << 1$ and $(2GM/w^2 r) << 1$, Eq.(15) reduces to the following well-known classical field equation

$$\mathbf{g} = -\nabla\phi\ . \tag{16}$$

4. Generalization of Newton's law of Gravitation

In this section, we shall generalize, in the framework of the CGA, the famous Newton's law of gravitation. So, in prior paper [1] we have already seen that the law of universal gravitational attraction

$$\mathbf{F} = -\frac{GMm}{r^3}\,\mathbf{r}\,, \tag{17}$$

is not really a single force in the common classical sense, but a resultant of two forces that make between them an extremely small angle, θ, especially when the test-body is in state of motion.The extreme smallness of that angle means that the $\mathbf{F}$ resultant force and its two components, namely, the static force $\mathbf{F}_\mathrm{S}$ and the dynamic force $\mathbf{F}_\mathrm{D}$ are almost in perfect superposition, and the resultant should be of the form $\mathbf{F} \equiv \mathbf{F}(r, v, \theta)$ as we shall soon seen. First, we have from Eqs.(1), (2) and (15) the following expression of the dynamic gravitational field (DGF):

$$\mathbf{\Lambda} = -\frac{v^2}{w^2}\nabla\phi - \frac{2GM}{w^2 r}\frac{d\mathbf{v}}{dt}\,, \tag{18}$$

And without loss of generality, let us neglecting the second term in the right hand side of (18) and multiplying the two sides of Eqs.(1) and (18) by the mass, m, of the moving test-body *B*, we get after addition the expression of the resultant force

$$\mathbf{F} = \mathbf{F}_\mathrm{S} + \mathbf{F}_\mathrm{D}\,. \tag{19}$$

Therefore, by using Eq.(19) and the well-known definition of the scalar product of two vectors

$$\mathbf{A}\cdot\mathbf{B} = \|\mathbf{A}\|\cdot\|\mathbf{B}\|\cos\theta\,, \tag{20}$$

where θ is between $\mathbf{A}$ and $\mathbf{B}$, which is, in our case, ranged between $\mathbf{F}_\mathrm{S}$ and $\mathbf{F}_\mathrm{D}$ or equivalently is between $\boldsymbol{\gamma}$ and $\mathbf{\Lambda}$. So, we have from (19) and (20)

$$\mathbf{F}^2 = \mathbf{F}_\mathrm{S}^2 + \mathbf{F}_\mathrm{D}^2 + 2\|\mathbf{F}_\mathrm{S}\|\cdot\|\mathbf{F}_\mathrm{D}\|\cos\theta\,, \tag{21}$$

from where we get

$$\|\mathbf{F}\| = \|\mathbf{F}_\mathrm{S}\|\cdot\left[1 + \mathbf{F}_\mathrm{D}^2\cdot\mathbf{F}_\mathrm{S}^{-2} + 2\|\mathbf{F}_\mathrm{D}\|\cdot\|\mathbf{F}_\mathrm{S}\|^{-1}\cos\theta\right]^{1/2}\,. \tag{22}$$

Again, by taking into account Eq.(1) and the above considerations, we have $\mathbf{F}_\mathrm{S} = -m\nabla\Phi$ and $\mathbf{F}_\mathrm{D} = -m\,(v/w)^2\nabla\Phi$, thus after substitution in (22), we get the expected expression

$$\mathbf{F} \equiv \mathbf{F}(r, v, \theta) = -\frac{k}{r^3}\left[1 + \left(\frac{v}{w}\right)^4 + 2\left(\frac{v}{w}\right)^2\cos\theta\right]^{1/2}\mathbf{r}\,, \tag{23}$$

where $k = GMm$ and $v_\mathrm{esc} \le w \le c_0$. Again, as it is easy to remark it, the expression of CGA-law of gravitation (23) is in excellent agreement with Einstein's claim, that's, " gravitation acts more strongly on a moving body than on the same body in case it is at rest." But why was the CGA-law (23) unknown? Because conceptually and physically, the famous Newton's law of gravitation (17) represents a limiting case for stationary or slowly moving material objects, and also because the angle, θ, ranged between $\mathbf{F}_\mathrm{S}$

and $\mathbf{F}_{\mathrm{D}}$ is generally very small, perhaps, that's why the physicists have used the classical form (17), without forgetting that GRT itself is founded on this same law with some modifications when velocities become relativistic and gravitational field becomes very strong; for this reason GRT reduces to Newton's gravity in the weak-field and low-velocity limit. Thus according to above considerations, Eq.(23) should be regarded as correction,
modification and generalization of the classical form (17). Although GRT does not consider gravity as a force properly speaking but interpreted as a curvature of space-time, however, it seems by applying the general force in Schwarzschild coordinate, Ridgely [4] was remarkably able to derive one expression of the gravitational force defined in the context of GRT

$$\mathbf{F} = -\frac{GMm}{r^2}\left[1 - \frac{2GM}{c^2 r}\right]^{-1/2}\mathbf{e}_r\,, \tag{24}$$

where $\mathbf{e}_r$ is a unit vector pointing in the r-coordinate direction. From Eq.(24), it is straightforward to see that there is a singularity, that is to say when $r \to (2GM/c^2)$, $\|\mathbf{F}\|$ becomes infinite. Such singularity/infinity is inherited from GRT which, as we know, is the realm of singularities! However, any coherent physical theory should prohibit the appearance of singularities/infinities in its formalism. Further, one of the most fundamental and profound distinction between a theory of physics and theory of mathematics is with respect to the concept of infinity. While in mathematics we can associate and attribute, in a perfectly logical and coherent way, the infinite value to the parameters, such associations are strictly meaningless when related to a theory of physics. And this is because in Nature nothing is infinite. All physical parameters of phenomena and objects of Nature are defined and characterized by finite values and only finite values. Nature cannot be described through infinite concepts and values as they are devoid of any meaning in the real physical world. Now, returning to Eq.(24) and writing it without singularity by supposing the quantity $(2GM/c^2 r)$ to be sufficiently less than unity, we obtain

$$\mathbf{F} = -\frac{GMm}{r^2}\left[1 + \frac{GM}{c^2 r}\right]\mathbf{e}_r\,. \tag{25}$$

Let us show that Eq.(25) is an important particular case of Eq.(23) when the moving test-body B of mass m evolving inside the vicinity of the main gravitational source A of mass M. Thus, by taking into account the above consideration and the definition (4), we get

$$\mathbf{F} \equiv \mathbf{F}(r, v, \theta) = -\frac{k}{r^3}\left[1 + 2\left(\frac{v}{c_0}\right)^2\cos\theta + \left(\frac{v}{c_0}\right)^4\right]^{1/2}\mathbf{r}\,, \tag{26}$$

Since in general $\theta \cong 0°$, thus when the test-body B orbiting the gravitational source A at the relative radial distance r with the orbital velocity $v = [GM/r]^{1/2}$, we obtain after substitution in (26), the following expression:

$$\mathbf{F} = -\frac{k}{r^3}\left[1 + \frac{GM}{c_0^2 r}\right]\mathbf{r}\,. \tag{27}$$

Since $k = GMm$ and $c_0 \equiv c$, therefore, Eq.(25) coincides perfectly with Eq.(27).

5. Role and Effects of Dynamic Gravitational Field

In terms of fields, the existence of the combined gravitational field (2) means for example that the Sun is really exerting on the Earth two gravitational fields, $\boldsymbol{\gamma}$ and $\boldsymbol{\Lambda}$, *via* $\mathbf{g}$ which is their resultant. The Newtonian gravity theory has ignored or missed the existence of $\boldsymbol{\Lambda}$. Therefore such an omission implies $\mathbf{g} = \boldsymbol{\gamma}$ and that's why the famous Newton's law of gravitation (17) is unable to explain qualitatively and quantitatively the well-observed extra-precession of Mercury perihelion, $43.11\ \text{arcsec/century}$. However, if historically, the GRT was capable of explaining the secular perihelion advance of Mercury this exploit is due in great part to the extra-field $\boldsymbol{\Lambda}$ or equivalently to the extra-force $\mathbf{F}_{\mathrm{D}}$ that may be deduced from Eq.(25) which as we know is, at the same time, a direct consequence of GRT for a test-body orbiting the main gravitational source and coincided perfectly with CGA-Eq.(27). Therefore, physically, the secular perihelion advance of Mercury and other planets of the Solar System is not caused by the curvature of space-time but causally is due to the couple $\langle \boldsymbol{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$ that acting on each planet as an extra field-force as we shall see. Now, returning to Eq.(18). Since, without loss of generality, we have already neglected the second term in right hand side of Eq.(18), accordingly the reduced expression of the dynamic gravitational (DGF), $\boldsymbol{\Lambda}$, takes the form

$$\boldsymbol{\Lambda} = -\ \nabla\phi\left(\frac{v}{w}\right)^2 \quad , \tag{28}$$

where $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$; $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ and $v_{\mathrm{esc}} \le w \le c_0$.

Therefore, after performing some differential and algebraic calculations, we get the expression of the $\boldsymbol{\Lambda}$-components:

$$\boldsymbol{\Lambda} : \begin{cases} \Lambda_x = -\dfrac{v^2}{w^2}\dfrac{GM}{r^2}\dfrac{x-x_0}{r} \\ \Lambda_y = -\dfrac{v^2}{w^2}\dfrac{GM}{r^2}\dfrac{y-y_0}{r} \\ \Lambda_z = -\dfrac{v^2}{w^2}\dfrac{GM}{r^2}\dfrac{z-z_0}{r} \end{cases} . \tag{29}$$

From (29), we arrive at the expression of the magnitude

$$\|\boldsymbol{\Lambda}\|^2 \equiv \Lambda^2 = (\Lambda_x^2 + \Lambda_y^2 + \Lambda_z^2) = \frac{G^2M^2}{r^4}\left(\frac{v}{w}\right)^4 . \tag{30}$$

Therefore

$$\Lambda = \pm\frac{GM}{r^2}\left(\frac{v}{w}\right)^2 . \tag{31}$$

Eq.(31) means that DGF, $\boldsymbol{\Lambda}$, may play a double role, that is to say, when perceived/interpreted as an extra-gravitational acceleration $(\Lambda > 0)$ or an extra-gravitational deceleration $(\Lambda < 0)$. More explicitly, we summarise the above considerations as follows: 1) When the velocity vector $\mathbf{v}$ of the moving test-body B is directed towards the gravitational source A, the DGF, $\boldsymbol{\Lambda}$, acting on B as an extra-gravitational acceleration of magnitude

$$\Lambda = \frac{GM}{r^2}\left(\frac{v}{w}\right)^2 . \tag{32}$$

2) And when the velocity vector $\mathbf{v}$ of the same moving test-body B is directed on the opposite side of the gravitational source A, the DGF, $\mathbf{\Lambda}$, acting on B as an extra-gravitational deceleration of magnitude

$$\Lambda = -\frac{GM}{r^2}\left(\frac{v}{w}\right)^2 . \tag{33}$$

Since in [1], we have already studied in detail the role and effects of DGF, therefore, in the present work we focus our interest only in the case $\Lambda > 0$, *i.e.,* when $\mathbf{\Lambda}$ playing the role of an extra-gravitational acceleration. So, let us consider a fixed observer in the inertial reference frame of the supposed stationary gravitational source A, and the moving test-body B is relatively situated far from A at a certain radial distance and supposing the following effects:

1) *Time contraction*: When the DGF, $\mathbf{\Lambda}$, playing the role of an extra-gravitational acceleration ($\Lambda > 0$) ,*i.e.*, when the test-body B starts to approach progressively the supposed stationary gravitational source A, and the velocity vector, $\mathbf{v}$, of B is directed towards A, the fixed observer should have the impression that the moving test-body B gains the time with respect to him, such 'time gain' is called-time contraction-. The amount of this temporal contraction is given by

$$\Delta t = \frac{1}{2}\Lambda v^{-1}t^2 = \frac{1}{2}\Lambda r^2 v^{-3}, \tag{34}$$

where t is the apparent duration of the relative motion of the test-body B.

2) *Space contraction*: Also, at the same time, the fixed observer should have the impression that the initial relative distance between A and B is in progressive contraction with respect to him, such-spatial shortening- is called, space contraction. The amount of this apparent variation in form of contraction is given by

$$\Delta r = \frac{1}{2}\Lambda t^2 = \frac{1}{2}\Lambda r^2 v^{-2} . \tag{35}$$

3) *Velocity increment*: Furthermore, the same fixed observer should notice that the relative velocity of test-body B is very slightly increasing with respect to him. Such small augmentation is called *velocity increment*. The amount of this *increment* is given by

$$\Delta v = \Lambda t = \Lambda r v^{-1}. \tag{36}$$

5.1. CGA-Effects in the Inner Solar System

The structural simplicity and the mathematical beauty that should characterize any modern physical theory do not fully suffice by themselves as intrinsic quality but also the well established theory should be characterized by its proper power of prediction and description of new effects without, of course, forgetting the old ones. Based on these lines of thought, the CGA as an alternative gravitational theory should be firstly tested locally, in the inner solar system (ISS) and secondly at global level, *i.e.*, in the outer solar system (OSS) which is our next purpose in this paper. As we know it, according to the CGA-formalism, the famous Newton's universal law of gravitation (17) is not really a single force in common classical sense, but a resultant $\mathbf{F}$ of two forces $\mathbf{F}_S$ and $\mathbf{F}_D$ that make between them a very small angle, θ. The smallness of that angle means that the resultant and its two components are almost in perfect superposition. Thus the main CGA-prediction is the existence of the dynamic gravitational field , $\mathbf{\Lambda}$, that is phenomenologically a sort of gravitational induction caused by the motion of test-body in the static gravitational field, $\boldsymbol{\gamma}$. In this sense, we said that the test-body is evolving in the combined gravitational field, $\mathbf{g}$, which is in fact the resultant of $\boldsymbol{\gamma}$ and $\boldsymbol{\Lambda}$. Furthermore, since $\boldsymbol{\Lambda}$ may be acted/behaved like an extra-gravitational acceleration or deceleration, therefore as an additional field $\boldsymbol{\Lambda}$ or force $\mathbf{F}_{\mathrm{D}}$, how the couple $\langle \boldsymbol{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$ can appear its effects in ISS?

In terms of field-force, in spite of their weak magnitude with respect to $\langle \boldsymbol{\gamma}, \mathbf{F}_{\mathrm{S}} \rangle$, the couple $\langle \boldsymbol{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$ has its proper effects in addition to those that have been already mentioned. These new-old additional effects are: the CGA-secular perigee precessions for the satellites and the CGA-secular perihelion precessions for the planets, particularly, when the DGF playing the role of an extra-gravitational acceleration. Einstein's GRT explains such secular celestial phenomena as a result of the local curvature of space-time around the Sun. However, like Newton's gravity theory, GRT does not take explicitly into account the existence of $\langle \boldsymbol{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$ as an additional gravitational field-force induced by the test-body during its motion in vicinity of the main gravitational source. Accordingly, in the context of CGA, we explain the above mentioned secular celestial phenomena as a direct consequence of $\langle \boldsymbol{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$.

5.2. Average magnitude of $\langle \boldsymbol{\Lambda}_i, \mathbf{F}_{\mathrm{D}_i} \rangle$ in ISS

Now, we wish to determine in the ISS the average magnitude of $\langle \boldsymbol{\Lambda}_i, \mathbf{F}_{\mathrm{D}_i} \rangle$ for each planet. The ISS gives us a very good opportunity to test the CGA because in such a system, the Sun plays the role of the principal gravitational source A of mass M, and each planet P_i may be separately played the role of the test-body B_i of mass m_i, where subscript (i = 1,2,3 ...9) denotes the order of each planet P_i in the ISS. For our purpose, Pluto is always considered as planet since for as long as this celestial body orbits the Sun like exactly the other planets. Thus according to the CGA, and in terms of field, the Sun as principal gravitational source is permanently exerting on each planet, P_i, during its orbital motion at average radial distance, r_i, with average orbital velocity, v_i, a certain DGF, $\boldsymbol{\Lambda}_i$, acting as an additional field. In such a case, the average radial distance between the planet and the Sun's centre of gravity is

$$r_i = \frac{r_i^{\min} + r_i^{\max}}{2}, \tag{37}$$

Since $r_i^{\min} = a_i(1-e_i)$ and $r_i^{\max} = a_i(1+e_i)$, where a_i and e_i are, respectively, the semi-major axis and orbital eccentricity of planet P_i. Hence, by substituting these relations in (37), we get immediately

$$r_i = a_i. \tag{38}$$

Further, for the case when the DGF plays the role of an extra-gravitational acceleration, we find after substitution in (32):

$$\Lambda_i = \frac{GM}{a_i^2}\left(\frac{v_i}{w}\right)^2, \tag{39}$$

Since we are dealing with the ISS, therefore we can, on average, consider each planet, P_i, being relatively in vicinity of the Sun. Consequently, according to the definition (4), we obtain from (39), for the case $w = c_0$:

$$\Lambda_i = \frac{GM}{a_i^2}\left(\frac{v_i}{c_0}\right)^2. \tag{40}$$

Furthermore, we have for the average orbital velocity the expression

$$v_i = \left[\frac{GM}{a_i} \right]^{1/2}, \tag{41}$$

hence by substituting (41) in (40), we get the important formula of the average magnitude of DGF as an extra-gravitational acceleration

$$\Lambda_i = \frac{1}{a_i} \left[\frac{GM}{c_0\, a_i} \right]^2 . \tag{42}$$

Or in terms of force, the Sun as principal gravitational source, is permanently acting on each planet a certain dynamic gravitational force, which behaves like an additional force. The average magnitude of this force is given by

$$F_{\mathrm{D}_i} = \frac{m_i}{a_i} \left[\frac{GM}{c_0\, a_i} \right]^2 . \tag{43}$$

Where m_i is the mass of planet P_i. Now, from the formulae (42) and (43), the predicted average magnitude $\left\langle \Lambda_i , F_{\mathrm{D}_i} \right\rangle$ of the couple $\left\langle \boldsymbol{\Lambda}_i , \mathbf{F}_{\mathrm{D}_i} \right\rangle$ for each planet is computed and listed in columns 4 and 5 of Table1; where for the values of the mass of the Sun and of the physical constants we take $M = M_\Theta = 1.9891 \times 10^{30}\ \mathrm{kg}$; $G = 6.67384 \times 10^{-11}\ \mathrm{m^3\, kg^{-1} s^{-2}}$ and $c_0 = 299792458\ \mathrm{m\,s^{-1}}$.

			Predicted CGA-effects	
Planet	a_i (m)	m_i (kg)	Λ_i $(\mathrm{m\,s^{-2}})$	F_{D_i} (N)
Mercury	57.92×10^9	3.286800×10^{23}	1.008547×10^{-9}	3.314893×10^{14}
Venus	108.25×10^9	4.870440×10^{24}	1.544892×10^{-10}	7.524304×10^{14}
Earth	149.60×10^9	5.972200×10^{24}	5.853115×10^{-11}	3.497236×10^{14}
Mars	227.95×10^9	6.394320×10^{23}	1.654486×10^{-11}	1.057931×10^{13}
Jupiter	778.30×10^9	1.899770×10^{27}	4.160406×10^{-13}	7.896628×10^{14}
Saturn	1.428×10^{12}	5.689152×10^{26}	6.729722×10^{-14}	3.828641×10^{13}
Uranus	2.870×10^{12}	8.724960×10^{25}	8.289647×10^{-15}	7.232684×10^{11}
Neptune	4.497×10^{12}	1.033848×10^{26}	2.154830×10^{-15}	2.227767×10^{11}
Pluto	5.900×10^{12}	1.254960×10^{22}	9.541699×10^{-16}	1.197445×10^{7}

Table 1. Above, column 1 gives the planet's name; column 2 gives the semi-major axis of each planet; column 3 gives the mass of each planet; columns 4 and 5 give, respectively, the values of Λ_i and F_{D_i} for each planet.

5.3. CGA-Formula for the Perigee and Perihelion precessions

After we have calculated the values of the average magnitude $\left\langle \Lambda_i, F_{\mathrm{D}_i} \right\rangle$ of the couple $\left\langle \mathbf{\Lambda}_i, \mathbf{F}_{\mathrm{D}_i} \right\rangle$ for each planet in ISS, at present, we will show that in despite of its weak average magnitude, the dynamic gravitational field $\mathbf{\Lambda}$ or equivalently the dynamic gravitational force $\mathbf{F}_{\mathrm{D}_i}$ is the main responsible for the observed secular perigee precessions for the satellites and the observed secular perihelion precessions for the planets in ISS. Hence, since the radial distance between the moving /orbiting test-body B and the main gravitational source A , undergoes a certain apparent variation with respect to the fixed observer in $A's$ inertial reference frame; therefore, with the help of the equation (35), we derive the expected CGA-formula as follows: Let the test-body B orbiting the main gravitational source A at a radial distance r with average orbital velocity v during each average orbital period P. According to the equation (35), under the influence of $\mathbf{\Lambda}$ as an additional gravitational field, the radial distance r undergoes a certain variation Δr when $\mathbf{\Lambda}$ playing the role of an extra-gravitational acceleration, *i.e.*, when the velocity vector $\boldsymbol{v}$ of B is directed towards the supposed stationary gravitational source A. This radial distance variation should induce a small secular advance of the perigee (if B is a satellite and A is a planet) or secular advance of the perihelion (if B is a planet and A is a star). The relative position of the celestial test-body moving along a Keplerian ellipse oscillates between a minimum radial distance of $r_{\min} = a(1-e)$ and a maximum radial distance of $r_{\max} = a(1+e)$ over one orbital revolution. If during this temporal interval $(t = P)$ the ellipse processes in its plan by a very small amount $\Delta\varphi$, the related variation Δr of the radial distance r would be approximately written as:

$$\Delta r = a\,\Delta\varphi\,, \tag{44}$$

From where we get

$$\Delta\varphi\,(\mathrm{rad/rev}) = \frac{\Delta r}{a}\,, \tag{45}$$

where a is the semi-major axis and (rad/rev) means that $\Delta\varphi$ is expressed in ‘radian per revolution’. Also, we have according to the equation (35) and the fact that $(t = P)$:

$$\Delta\varphi = \frac{1}{2}\frac{\Lambda}{a}P^2\,. \tag{46}$$

Here P is the average orbital period expressed in seconds. Further, since here we are dealing with the average orbital parameters, thus according to (42), and by omitting the subscript ‘i’, we can finally obtain, after substituting (42) into (46), the expected CGA-formula:

$$\Delta\varphi = \frac{1}{2}\left[\frac{GMP}{c_0\,a^2}\right]^2, \tag{47}$$

where M is the mass of the principal gravitational source. Also, we can express (46) in terms of the magnitude of the dynamic gravitational force, since $\Lambda = F_{\mathrm{D}}/m$, where m is the mass of the orbiting test-body, thus after substitution in (46), we get

$$\Delta\varphi = \frac{1}{2}\frac{F_{\mathrm{D}}P^2}{m\,a}\,. \tag{48}$$

The CGA-formulae (47) and (48) show us that $\left\langle \mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}} \right\rangle$ are explicitly responsible for the mentioned secular orbital precessions. Besides to what was already mentioned, it is worthwhile to note that, phenomenologically, during its orbital motion, the celestial test-body undergoes a certain *apparent* change in its orbital period and orbital velocity caused by the DGF when it behaves like an extra-gravitational acceleration. Hence, according to the equations (34) and (36), the change in orbital period and orbital velocity are, in the case of orbital motion, of the form

$$\Delta P = \frac{1}{2}\,\Lambda\, v^{-1} P^2 , \tag{49}$$

$$\Delta v = \Lambda\, P . \tag{50}$$

More explicitly, after omitting the subscript *'i'* in (44) and by taking into account the expression of the average orbital velocity $v = 2\pi\, a\, P^{-1}$, we can rewrite the above formulae as follows:

$$\Delta P = \frac{P}{4\pi}\left[\frac{GMP}{c_0\, a^2}\right]^2 , \tag{51}$$

$$\Delta v = \frac{P}{a}\left[\frac{GM}{c_0\, a}\right]^2 . \tag{52}$$

5.4. Calculation of the secular perigee precession of the Moon

Without doubt, one of the most important celestial bodies, the Moon, is literally at the Earth's doorstep. The Moon is important for what it can tell us about, for example, the formation and evolution of the solar system (SS). It is important because it can serve as a veritable celestial laboratory enabling us to understand physical processes that take place on the Moon as well as on other similar SS-bodies and also to test some new gravity theories because it is natural to think of utilizing planetary satellites moving at average radial distance quite small in comparison with the semi-major axes of the planets' orbits; and indeed, De Sitter [5,6,7] chose our Moon as a test-object as long ago as 1916. Although he was initially concerned with determining the modification of the Moon's orbit resulting from the combined attraction of the Earth and the Sun under Einstein's GRT, it was found that the modification imposed by Einstein's theory on the gravitational field of the Earth alone resulted in an advance of the secular lunar perigee of 0.06 arcsec/cy [8]; where 'arcsec/cy' is the abbreviation for arc second per century. Hence, the correct calculation of the secular lunar perigee precession represents for any alternative gravity theory a fact of an extreme significance. In what follows we perform this calculation with the help of the CGA-formula (47). Since in the system Earth-Moon, the Earth playing the role of principal gravitational source A and the Moon has the role of test- body B. In the case of the Moon, we have $a = 3.844\times10^8\ \mathrm{m}$, $P = 27\,\mathrm{d}\ 7\,\mathrm{h}\ 43\,\mathrm{min} = 27.32\ \mathrm{d} = 2.360580\times10^6\ \mathrm{s}$, while for the values of the mass of the Earth and of the physical constants, we take $M = M_\oplus = 5.9722\times10^{24}\ \mathrm{kg}$, $G = 6.67384\times10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}}$, $c_0 = 299\,792\,458\ \mathrm{m\,s^{-1}}$. After substituting all these quantities in (47), we find

$$\Delta\varphi = 2.255\times10^{-10}\ \text{rad/rev} = 2.255\times10^{-10}\times\frac{180}{\pi}\times3600\times\left(\frac{36525}{27.32}\right) = 0.062\ \text{arcsec/cy}. \tag{53}$$

This is in good agreement with the value found by De Sitter.

5.5. Calculation of the secular perihelion precession of the Planets

After we have applied the CGA-formula (47) to calculate the secular perigee precession of the Moon and we have got the numerical value (53) which is in good accordance with that found by De Sitter, thus at present, we focus our attention on the secular perihelion precession of planets in ISS. Furthermore, among other things, our main interest is to show more conclusively the applicability and generality of the (47) in ISS. Since in the SS, the Sun playing the role of principal gravitational source of mass $M = M_{\Theta} = 1.9891\times10^{30}$ kg and each planet has the role of celestial test-body, thus by inserting the subscript (i = 1,2,3 ... 9) and replacing M with M_{Θ} in (47), we get

$$\Delta\varphi_i = \frac{1}{2}\left[\frac{GM_{\Theta}P_i}{c_0\, a_i^2}\right]^2 . \tag{54}$$

So, based on (54), we can construct the following Table 2 of CGA-secular perihelion precession for each planet. Thus it what follows we perform these calculations exactly as we have previously done for the Moon.

Planet	a_i (m)	P_i (d)	CGA- Predicted values $\Delta\varphi_i$ (arcsec/cy)	observed values $\Delta\varphi_i^{obs}$ (arcsec/cy)
Mercury	57.92×10^9	87.97	43.1198	43.1100
Venus	108.25×10^9	224.70	9.0270	8.4000
Earth	149.60×10^9	365.25	4.0227	5.0000
Mars	227.95×10^9	686.97	1.4035	1.3624
Jupiter	778.30×10^9	4332.60	0.0651	0.0637
Saturn	1.428×10^{12}	10759.20	0.0142	0.0140
Uranus	2.870×10^{12}	30686.00	0.0025	- - - (a)
Neptune	4.497×10^{12}	60189.00	0.0008	- - - (a)
Pluto	5.900×10^{12}	90472.00	0.0004	- - - (a)

Table 2. Above, column 1 gives the planet's name; column 2 gives the semi-major axis of each planet; column 3 gives the average orbital period of each planet; column 4 gives the CGA-predicted values of $\Delta\varphi_i$ for each planet and column 5 gives the observed values.

Notes: [a] Because their long orbital duration covering at least two human lifetimes, no data is currently available covering one full orbital revolution for Neptune and Pluto hence there is not yet any observational values for the precession of their perihelia.

From the table 2, we can affirm that the CGA-predicted secular perihelion advance for each planet of the ISS is, generally, in good agreement with the observed value.

6. CGA-Effects in the Outer Solar System

Eclipsing binary star systems are a great stellar laboratory particularly for testing the gravity theories *via* the study of the apsidal motions. Before the advent of the CGA, the apsidal motion is generally explained as follows: when the gravitational field of a star differs from that of a Newtonian point, the orbit of its companion will deviate from a Keplerian orbit. To lowest order, a perturbation to the r^{-1}-gravitational potential causes the periastron to rotate. This is the origin of apsidal motion. Usually, there are primarily three effects that cause deviation from r^{-1}-gravitational potential: the general relativistic correction to Newtonian gravity theory, the quadrupole moment that arises due to the rotational distortion of a star, and the quadrupole moment due to tidal distortion. The first two effects are relatively easy to calculate and are well understood. The third effect, the modification of the gravitational potential due to tidal distortion displays more complex behavior. The derivation of the formula for apsidal motion due to classical (Newtonian) effects was first worked out by Cowling (1938) and Stern (1939). Also, before the establishment of the CGA, it has been argued for a long-time that in the vast majority of close binary systems, the apsidal motion is dominated by the classical and Relativistic effects. Hence, the observed rate of apsidal motion is due to the contribution of two terms: a classical term $\dot{\omega}_{\mathrm{CL}}$ as well as the general relativistic term which, according to Levi-Cevita [9] and Kopal
[10], is of the form

$$\dot{\omega}_{\mathrm{GR}}\,(\mathrm{deg/yr}) = 9.2872\times10^{-3}\left(\frac{2\pi}{P}\right)\frac{(M_1+M_2)^{2/3}}{(1-e^2)}, \tag{55}$$

where M_1, M_2 are in solar mass and P is in days. In this sense, the observed apsidal motion rate should be

$$\dot{\omega}_{\mathrm{OBS}} = \dot{\omega}_{\mathrm{CL}} + \dot{\omega}_{\mathrm{GR}} = \dot{\omega}_{\mathrm{CL+GR}}\,. \tag{56}$$

However, it has been pointed out for a long-time the existence of a certain notable discrepancy between the expected theoretical value, $\dot{\omega}_{\mathrm{CL+GR}}$, and the observed value, $\dot{\omega}_{\mathrm{OBS}}$, of the periastron advance of several eclipsing binary star systems likes, *e.g.,* DI Herculis [11]; AS Camelopardalis [12];V1143 Cygni [13,14]; V459 Cassiopeia [15,16]. Guinan and Maloney [11] have argued that alternative theories of gravitation may be needed to explain the discrepancy. In the absence of a reasonable classical explanation for this discrepancy in the observed apsidal motions, there exists the possibility that the pointed out discrepancy is a sure signal of the limit of Einstein's GRT, that's why, *e.g.*, Moffat [17,18,19] proposed a nonsymmetric gravity theory (NGT).

6.1. CGA-Apsidal Motion

Let us consider a hypothetical eclipsing binary system $\{A, B\}$ of masse M_A and M_B $(M_B \le M_A)$ evolving in the mutual combined gravitational field, $\mathbf{g} = \boldsymbol{\gamma} + \boldsymbol{\Lambda}$. The system comprises two stars A and B closely moving in elliptical orbits around their common center of mass, as illustrated below in the Figure 1.
Each star moves in its orbit according to Kepler’s laws, at all times the two stars are found on opposite sides of a line passing through their common center of mass.

How the CGA-apsidal motion occurs? Since the orbits of the two stars A and B are elliptical, the two are closer together at some times than at others, so that the dynamic gravitational field, $\boldsymbol{\Lambda}$, or equivalently the dynamic gravitational force, $\mathbf{F}_{\mathrm{D}}$, alternately strengthens at periastron and weaken at apastron. In view of the fact that $\mathbf{F}_{\mathrm{D}}$ is physically an extra-gravitational force, therefore, its action as an additional force causes the orbit of the system to advance. The orbit of the system appears to rotate with time.

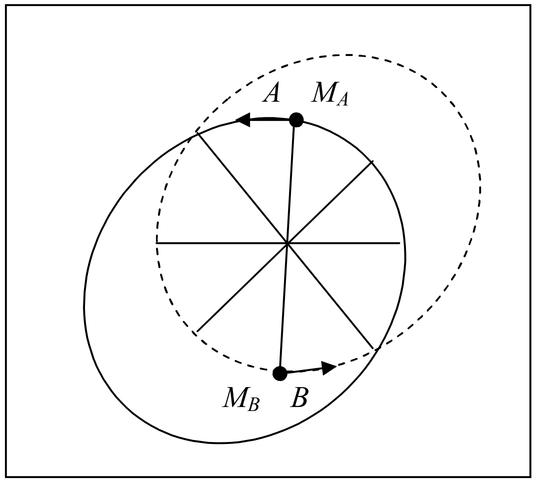

Figure 1: The orbit of the hypothetical binary star system $\{A, B\}$ shown from above the orbital plane. The solid line represents the orbit of the primary (A) component and the dashed line the orbit of the secondary (B). The lines from the common center of mass towards the orbits indicate the relative positions of the periastron. The big dots indicate the relative positions of the stars at time of mid primary eclipse.

The permanent action of $\mathbf{F}_\mathrm{D}$ prevents the orbit to be closed ellipse, but a continuous elliptical arc whose point of closest approach (periastron) rotates with each orbits. In fact, the rotation of the system's periastron is very analogous to the advance of the perihelion of the planets in their orbits.

6.1.1. Equations of CGA-Apsidal motion for Binary Star Systems

It is worthwhile to note that the expression of the formulae (42), (43), (47), (51) and (52) only hold for the motion of planets about the Sun. In this case, the mass ratio, $q = M_B / M_A$, of system $\{A, B\}$ is very comparable to zero ($q \cong 0$), that's why we have supposed that the Sun is at rest and it is an inertial reference frame. Further, the orbital eccentricity, e, does not occur in the expression of these formulae because we have taken $e^2 \cong 0$, such approximation is due to the great mean distance of the planets from the Sun. however, the above considerations are not always legitimate particularly for the eclipsing binary star system *i.e.*, when A and B playing the role of two stars of masses M_A and M_B, which are gravitationally linked. Contrary to the Sun-planet system, the study of eclipsing binary star systems is not easy task because the mass ratio, q, is not always less than unity but sometimes is (approximately) equal to unity and also the distance separating the two stars is more often ranged between the Sun's radius $(R_\Theta = 695508\ \mathrm{km})$ and AU, hence, that's why the orbital eccentricity of the system should be taken into consideration whatever its numerical value. Therefore, for the case when $q < 1$, the star A of mass M_A is the main gravitational source and the second star B of mass M_B playing the role of test-body, and when $q = 1$, the two stars may be mutually played the role of the main gravitational source. Consequently, in the context of the CGA, the knowledge of q with enough accuracy is a fundamental condition because this mass ratio is an essential element for the function $f(e,q)$ called: orbital eccentricity-mass ratio function, and for the scalar parameter M which having the physical dimensions of mass; therefore the two scalar quantities $\{ f(e,q)\,; M \}$ should be taken into account when we would generalize the formulae (42), (43), (47), (51) and (52) to the eclipsing binary star systems. Hence, for the seek of simplicity, accuracy and generality, the cited formulae should very slightly modified after omitting the subscript 'i' and when we take the usual notation for the apsidal motion rate, $\dot{\omega}$, the formulae (42), (43), (47), (51) and (52) become, respectively, as follows:

$$\Lambda = \frac{1}{a}\left[\frac{GM}{c_0\, a}\right]^2. \tag{57}$$

$$F_{\mathrm{D}} = \frac{M_B}{a}\left[\frac{GM}{c_0\, a}\right]^2. \tag{58}$$

$$\dot{\omega}_{\mathrm{CGA}} = \frac{f(e,q)}{2}\left[\frac{GM\,P}{c_0\, a^2}\right]^2, \tag{59}$$

$$\Delta P = \frac{P}{4\pi}\left[\frac{GM\,P}{c_0\, a^2}\right]^2, \tag{60}$$

$$\Delta v = \frac{P}{a}\left[\frac{GM}{c_0\, a}\right]^2 . \tag{61}$$

Where $f(e,q)$ is the orbital eccentricity-mass ratio function and M is a scalar parameter having the dimensions of mass, and both are defined as follows:

$$f(e,q) = \begin{cases} 1, \text{ if } e << \frac{1}{4} \\ (1+e)^{4q} + \frac{q}{6}(1-e)\left[11 - 2e + e^2\right], \text{ if } e < \frac{1}{4} \\ 1 + \frac{qe}{2}\left[e - \frac{q}{4}e^3 + \frac{q}{16}e^5\right], \text{ if } \frac{1}{4} \le e < \frac{1}{2} \\ 1 + qe\left[e^3 + qe + q^3\right] \pm \frac{1}{2}q, \text{ if } \frac{1}{2} \le e < 1 \\ (1-e^2)^{q-1}, \text{ if } q << 1 \end{cases}, \tag{62}$$

$$M = \begin{cases} M_A, \text{ if } q < 1 \\ M_A + M_B, \text{ if } q = 1 \end{cases}. \tag{63}$$

Thus, the generalized expressions (57-61) are the CGA-formulae that permit us to investigate the CGA-effects in eclipsing binary star systems and in binary pulsars as we will see soon. Also, the CGA-effects are in fact post-Keplerian/Newtonian effects since they concern at the same time the orbital parameters and the gravitational field-force. Before listing in the Table 2 the expected CGA-effects for some well-known eclipsing binary star systems, we prefer to beginning with the investigation of CGA-effects in DI Herculis and AS Camelopardalis in order to make easy the comprehension of the process of calculation *via* CGA-formulae.

6.1.2. AS Camelopardalis

AS Cam is an eclipsing binary star system. Like DI Her and a few other systems, AS Cam is an important test case for gravity theories. Accurate determinations of the orbital and stellar parameters of AS Cam have been made by Hilditch [20,21] and Khalliulin & Kozyreva [22] that permit the expected classical and relativistic contributions to the apsidal motion to be determined reasonably well:

$$\dot{\omega}_{\mathrm{CL}} = 35.80 \ \mathrm{deg/cy}, \tag{64}$$

and

$$\dot{\omega}_{\mathrm{GR}} = 8.50 \ \mathrm{deg/cy}. \tag{65}$$

Maloney *et al*., [12] have gathered all the published timings of primary and secondary minima, and have reinforced these with eclipse timings from 1899 to 1920 obtained from the Harvard plate collection. Least-square solutions of the eclipse timings extending over an 80 yr interval yield a smaller than expected apsidal motion rate of

$$\dot{\omega}_{\mathrm{OBS}} = 15 \ \mathrm{deg/cy}, \tag{66}$$

in agreement with that found by [22] from a short set of data. As we can remark it, the observed apsidal motion rate (66) for AS Cam is about one-third that theoretically expected from the combined classical and relativistic effects:

$$\dot{\omega}_{\mathrm{CL+GR}} = 44.30 \ \mathrm{deg/cy}. \tag{67}$$

Thus, AS Cam joins DI Her in having an observed apsidal motion rate significantly less than that predicted from Newtonian and Einsteinian gravity theory. Here we shall see that there are two main causal sources of this profound disagreement which are, respectively, the high over estimation of classical contribution to the apsidal motion and the complete ignorance of the existence of the couple $\langle \mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$.However, when we neglected or minimize the evoked classical contribution and applying the CGA-formalism, we shall find a CGA-apsidal motion rate, $\dot{\omega}_{\mathrm{CGA}}$, compared to $\dot{\omega}_{\mathrm{GR}}$ and their combination, $\dot{\omega}_{\mathrm{CGA+GR}}$, yields a value in good agreement with the observed rate (66). To this end, we have according to [12] the following orbital and stellar parameters of AS Cam: $e = 0.1695$; $P = 3.430$ days; $a = 17.20\ R_{\Theta}$; $M_A = 3.3\ M_{\Theta}$; $M_B = 2.5\ M_{\Theta}$; $q = 0.7575$. Since $e < 1/4$ and $q < 1$, therefore the eccentricity-mass ratio function (62), scalar parameter (63) take, respectively, the form :

$$f(e,q) = (1+e)^{4q} + \frac{q}{6}(1-e)\left[11 - 2e + e^2\right], \quad M = M_A ,$$

and the formula (59) becomes,

$$\dot{\omega}_{\mathrm{CGA}} = \frac{f(e,q)}{2}\left[\frac{GM_A P}{c_0\, a^2}\right]^2$$

-*Numerical Application*: We have $f(e,q) = 2.7279$; $GM_A P = 1.30\times10^{26}\ \mathrm{m^3\,s^{-1}}$; $c_0\, a^2 = 4.3\times10^{28}\ \mathrm{m^3\,s^{-1}}$; and by substituting in the above formula, we get

$$\dot{\omega}_{\mathrm{CGA}} = 7.60 \ \mathrm{deg/cy}. \tag{68}$$

This result means that the CGA-effects contribute to the total observed apsidal motion rate at 50.66 % and consequently if we neglect or minimize the classical contribution, we find that the CGA-contribution completes the GR-effects and in this case, the theoretical expected apsidal motion rate should be of the form:

$$\dot{\omega}_{\mathrm{CGA+GR}} = 7.60 \text{ deg/cy} + 8.50 \text{ deg/cy} = 16.10 \text{ deg/cy}. \tag{69}$$

This is in good agreement with the observed value (66).

6.1.3. DI Herculis

Again, we are returning to the famous eclipsing binary star system DI Her because of its historical and astrophysical importance. For the past three decades, and until recently, there has been a serious discrepancy between the observed and theoretical values of the apsidal motion rate of DI Her, which has even been interpreted occasionally as a possible failure of GRT since the GR-contribution ($\dot{\omega}_{\mathrm{GR}} = 2.34$ deg/cy) is dominant for DI Her. Now, accuracy measured apsidal motion rate of

$$\dot{\omega}_{\mathrm{OBS}} = 1.04 \text{ deg/cy}, \tag{70}$$

determined from new analysis of numerous times of primary and secondary eclipse [23]. As it has been cited, the most remarkable feature of DI Her is that its observed apsidal motion rate (70) is significantly smaller than that theoretically predicted by classical and GR-contribution. The total predicted rate is

$$\dot{\omega}_{\mathrm{CL+GR}} = 4.27 \text{ deg/cy}. \tag{71}$$

However, recent observations of the Rossiter-McLaughlin effect [24,25], which was interpreted by Albrecht *et al.*, [26] as the reason for the anomaly is that the rotational axes of the stars and the orbital axis are misaligned, which changes the predicted rate of precession. Thus, according to [26] the misalignment causes *retrograde* apsidal motion rate, $\dot{\omega}_{\mathrm{RG}}$, of

$$\dot{\omega}_{\mathrm{RG}} = -\,2.14 \text{ deg/cy}, \tag{72}$$

and by taking into account the total predicted rate (71), we get the net theoretical precession rate of

$$\dot{\omega}_{\mathrm{NET}} = \dot{\omega}_{\mathrm{CL+GR}} + \dot{\omega}_{\mathrm{RG}} = 2.13 \text{ deg/cy}. \tag{73}$$

However, it seems even with the introduction of the retrograde apsidal motion rate (72) the discrepancy persistes since the net rate of precession (73) amounts to 200 % or more. At present, we will see that the CGA, as an alternative gravity theory, should be able to handle this problem very well and without introducing the retrograde apsidal motion rate (72), that is only by applying the CGA-formalism, we will obtain a value of CGA-apsidal motion rate, $\dot{\omega}_{\mathrm{CGA}}$, exactly comparable to the observed rate (70). So to this aim, we have according to [26] the following orbital and stellar parameters: $e = 0.489$; $P = 10.55$ days; $a = 43.12\ R_{\Theta}$; $M_A = 5.15\ M_{\Theta}$; $M_B = 4.52\ M_{\Theta}$; $q = 0.8776$. Since $e \approx 1/2$ and $q < 1$, therefore, the eccentricity-mass ratio function (62), scalar parameter (63) and the formula (59) take, respectively, the form :

$$f(e,q) = 1 + qe\left[e^3 + qe + q^3\right] + \frac{1}{2}q, \quad M = M_A\ ,$$

and

$$\dot{\omega}_{\mathrm{CGA}} = \frac{f(e,q)}{2}\left[\frac{GM_A P}{c_0\, a^2}\right]^2 .$$

-*Numerical Application*: We have $f(e,q) = 1.963211$; $GM_A P = 6.23 \times 10^{26}\ \mathrm{m^3\,s^{-1}}$; $c_0\, a^2 = 2.7 \times 10^{29}\ \mathrm{m^3\,s^{-1}}$; and after substitution in the above formula, we obtain

$$\dot{\omega}_{\mathrm{CGA}} = 5.22620 \times 10^{-6}\ \text{rad/rev} = 5.22620 \times 10^{-6} \times \frac{180}{\pi} \times 3600 \times \left(\frac{36525}{10.55}\right) = 1.03720\ \mathrm{deg/cy}. \qquad (74)$$

This is in excellent agreement with the observed value of $\dot{\omega}_{\mathrm{OBS}} = 1.04\ \mathrm{deg/cy}$ at 99.73 % !. This result shows us that the CGA-contribution for DI Her is dominant. For the other CGA-effects, *viz*., the average magnitude of the dynamic gravitational force; exerted by the main gravitational source A of mass M_A on the orbiting test-body B of mass M_B; the average change in orbital period and orbital velocity of system $\{A, B\}$. Since $q < 1$, thus the formulae (58), (60) and (61) become, respectively:

$$F_{\mathrm{D}} = \frac{M_B}{a}\left[\frac{GM_A}{c_0\, a}\right]^2, \qquad \text{(i)}$$

$$\Delta P = \frac{P}{4\pi}\left[\frac{GM_A P}{c_0\, a^2}\right]^2, \qquad \text{(ii)}$$

$$\Delta v = \frac{P}{a}\left[\frac{GM_A}{c_0\, a}\right]^2. \qquad \text{(iii)}$$

Direct numerical application gives us the following values of the expected CGA-effects: $F_{\mathrm{D}} = 1.730 \times 10^{24}\ \mathrm{N}$; $\Delta P = 3.8640 \times 10^{-1}\ \mathrm{s}$; $\Delta v = 1.75160 \times 10^{-1}\ \mathrm{m\,s^{-1}}$.

Now, we conclude the investigation of CGA-effects in noncompact stellar objects by selecting four other well-known eclipsing binary star systems: V1143 Cygni, V541 Cygni, V526 Sagittarii and V459 Cassiopeia. Their orbital, stellar parameters and CGA-effects are listed in Tables 3 and 4, respectively.

System	P (d)	e	a/R_Θ	M_A/M_Θ	M_B/M_Θ	Ref.
V 1143 Cyg	7.640	0.540	22.67	1.355	1.327	a
V 541 Cyg	15.340	0.479	43.82	2.240	2.240	b
V 526 Sgr	1.920	0.2194	10.27	2.270	1.680	c
V459 Cas	8.460	0.0244	27.67	2.020	1.960	d, e

Table 3. Orbital and Stellar Parameters of 4 selected Eclipsing Systems

Ref.: a) Albrecht *et al*. [26]; b) Lacy [27] ; c) Lacy [28] ; d) Lacy *et al*., [15] ; e) Dariush [16]

System	Predicted Values of the CGA- effects				
	$\dot{\omega}_{\mathrm{OBS}}$ (deg/yr)	$\dot{\omega}_{\mathrm{CGA}}$ (deg/yr)	F_{D} (N)	ΔP (s)	Δv (m/s)
V 1143 Cyg	3.370×10^{-2}	3.200×10^{-2}	9.497×10^{23}	5.225×10^{-1}	2.375×10^{-1}
V 541Cyg	0.600×10^{-2}	0.600×10^{-2}	6.193×10^{23}	8.455×10^{-1}	1.842×10^{-1}
V 526 Sgr	2.454	0.164	9.263×10^{24}	1.410×10^{-1}	4.598×10^{-1}
V459 Cas	6.045×10^{-2}	1.500×10^{-2}	1.698×10^{24}	7.040×10^{-1}	3.185×10^{-1}

Table 4. Predicted values of the CGA-effects

7. Compact Stellar Objects as Test of CGA

After we have investigated the CGA-effects in the noncompact stellar objects like the eclipsing binary star systems by showing that in addition to classical and relativistic effects, there are new other effects caused by the couple $\langle \mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$. For example, the computed CGA-apsidal motion rate, $\dot{\omega}_{\mathrm{CGA}}$, is in some cases in excellent agreement with the observed ones and sometimes it is comparable to the GR-rate, $\dot{\omega}_{\mathrm{GR}}$. Also, CGA and GR-contribution may be together played the role of mutual complementarity like, *e.g.*, the case of AS Cam when we have omitted the CL-contribution; consequently, the cited discrepancy was immediately concealed.

At present, we wish to push forward the frontiers of application of the CGA to investigate the same CGA-effects in the compact stellar objects like, *e.g.*, the white dwarfs, neutron stars and pulsars. That is to say, we test the CGA in critical domain where the gravitational field is extremely strong. Here, we focus our main interest in some well-known binary pulsars (pulsars and their companions). But first what's a pulsar?

Pulsar (pulsating star) is a rapidly rotating neutron star that emits a radio beam that is eventually powered by the pulsar's rotational energy and that is centered on the magnetic axis of the neutron star. As the magnetic axis and the hence the beam are inclined to the rotation axis, the pulsar acts as a cosmic lighthouse, and a pulsar appears a pulsating radio source. The moment of inertia and the stored rotational energy of pulsars are large, so that in particular the fast rotating millisecond pulsars deliver a radio "tick" per rotation with an extraordinary precision that rivals even the best atomic clocks on Earth! As they concentrate an average of 1.4 solar mass on a diameter of only about 20 km, pulsars are exceedingly dense and compact, that's why they representing the known densest matter in the observable universe. The resulting gravitational field near the pulsar' surface is large, thus enabling strong-field tests of gravity theories. Furthermore, pulsars and their orbiting companions are generally compact enough that their motion can be treated as that of two point masses. Thus in the context of CGA, we can logically consider each pulsar as the main gravitational source A of mass M_A and each orbiting companion as the test-body B of mass M_B. Consequently, the causal source of CGA-effects in the binary pulsar systems is exactly of the same nature as for ordinary (noncompact) eclipsing binary star systems. Therefore, the combined gravitational field, $\mathbf{g} = \boldsymbol{\gamma} + \mathbf{\Lambda}$, becomes more and more strong as the pulsar and its companion are so close together that an ordinary star like the Sun could not fit in their orbits. As result, the couple $\langle \mathbf{\Lambda}, \mathbf{F}_{\mathrm{D}} \rangle$ should have its intensity amplified drastically. That's why, *e.g.*, the value of the CGA-apsidal motion rate of binary pulsar systems should be more important than that of ordinary eclipsing binary star systems. Like before, that is when we have studied the latter systems, the determination of the CGA-effects in binary pulsars should show us, among other things, that the usual relativistic interpretation of gravity as a deformation of space-time is not a physical reality but a pure topological property of Riemann geometry which is conceptually non-Euclidean. We have selected some well-known binary pulsars in order to show the importance of GCA as an alternative gravity theory capable of studying the compact stellar objects *via* the investigation of the CGA-effects in such systems. Before summarizing our results in Table 5, we prefer to start with the study of the famous binary pulsars PRS B1913+16, binary pulsar PRS B1534+12 and the remarkable double binary pulsar PSR J0737-3039.

7.1. Binary pulsar PSR B 1913+16

The PRS B1913+16 is the first binary pulsar discovered in 1974 by Russell Hulse and Joseph Taylor [29]. It is since then, considered as an ideal celestial laboratory providing decisive tests of a wide class of gravity theories because the extreme conditions are well available in such massive and compact astrophysical objects, specifically, their strong gravitational field and rapid motion. Thus the investigation of the CGA-effects in the binary pulsar systems using the same CGA-formalism as for the case of the eclipsing binary star systems, is all the more impressive considering that, in contrast to some alternative gravity theories, CGA has no 'freedom' to adjust its predictions. It is highly constrained by its inadjustable formalism, that is to say, the CGA-equations do not contain adjustable parameters. Let us now investigate the CGA-apsidal motion and other CGA-effects in PSR B 1913+16. We have according to Weisberg and Taylor [30] the following orbital and stellar parameters of PSR B 1913+16: $e = 0.6171$; $P = 0.322997\,\text{d}$; $a = 1.950100\times10^{9}\,\text{m}$; $\dot{\omega}_{\text{OBS}} = 4.226595$ deg/yr ; $M_A = 1.4414\, M_\Theta$; $M_B = 1.3867\, M_\Theta$; $q = 0.9620$. Since $e > 1/2$ and $q \approx 1$; therefore the eccentricity-mass ratio function (62), scalar parameter (63) and the formula (59) take, respectively, the form :

$$f(e,q) = 1 + qe\left[e^3 + qe + q^3\right] - \frac{1}{2}q\,, \quad M = M_A + M_B\ ,$$

and

$$\dot{\omega}_{\text{CGA}} = \frac{f(e,q)}{2}\left[\frac{G(M_A + M_B)P}{c_0\, a^2}\right]^2 .$$

-*Numerical Application*: We have $f(e,q) = 1.539441$; $G(M_A + M_B)P = 1.047705\times10^{25}\ \text{m}^3\,\text{s}^{-1}$; $c_0\, a^2 = 1.140077\times10^{27}\ \text{m}^3\,\text{s}^{-1}$. By substituting in the above formula, we get

$$\dot{\omega}_{\text{CGA}} = 4.213832\ \text{deg}/\text{yr}. \tag{75}$$

This is in excellent agreement with the observed value at 99.70 %. For the other CGA-effects, the formulae (58), (60) and (61) take for the case $q \approx 1$ the following expressions, respectively:

$$F_{\text{D}} = \frac{M_B}{a}\left[\frac{G(M_A + M_B)}{c_0\, a}\right]^2 , \tag{iv}$$

$$\Delta P = \frac{P}{4\pi}\left[\frac{G(M_A + M_B)P}{c_0\, a^2}\right]^2 , \tag{v}$$

$$\Delta v = \frac{P}{a}\left[\frac{G(M_A + M_B)}{c_0\, a}\right]^2 . \tag{vi}$$

Direct numerical application gives us the following values of the expected CGA-effects: $F_{\mathrm{D}} = 5.831340 \times 10^{26}\ \mathrm{N}$; $\Delta P = 1.877100 \times 10^{-1}\ \mathrm{s}$; $\Delta v = 5.902849\ \mathrm{m\,s^{-1}}$.

7.2. Binary pulsar PSR B 1534+12

PRS B1534+12 had been discovered in1990 by Wolszczan [31]. A discussion of the relativistic effects in this binary system, and the resulting updated tests of GRT have been presented by Stairs *et al*.,[32]. Let us now determine the CGA-apsidal motion rate and the other CGA-effects in PRS B1534+12. We have, according to Nice *et al*.,[33], the following orbital and stellar parameters of PRS B1534+12: $e = 0.274$; $P = 0.420\,\mathrm{d}$; $a = 2.281697 \times 10^{9}\ \mathrm{m}$; $\dot{\omega}_{\mathrm{OBS}} = 1.756\ \mathrm{deg/yr}$; $M_A = M_B = 1.34 M_\Theta$; $q = 1$. In view of the fact that $e > 1/4$ and $q = 1$, therefore the eccentricity-mass ratio function (62), scalar parameter (63) and formula (59) take, respectively, the form:

$$f(e,q) = 1 + \frac{qe}{2}\left[e - \frac{q}{4}e^3 + \frac{q}{16}e^5\right],\ M = M_A + M_B,$$

and

$$\dot{\omega}_{\mathrm{CGA}} = \frac{f(e,q)}{2}\left[\frac{G(M_A + M_B)P}{c_0\, a^2}\right]^2$$

Numerical application: we have $f(e,q) = 1.036846$; $G(M_A + M_B)P = 1.291011 \times 10^{25}\ \mathrm{m^3\,s^{-1}}$; $c_0\, a^2 = 1.560761 \times 10^{27}\ \mathrm{m^3\,s^{-1}}$. By substituting all these values in the above formula, we obtain:

$$\dot{\omega}_{\mathrm{CGA}} = 1.767398\ \mathrm{deg/yr}. \tag{76}$$

This is in good agreement with the observed value. For the other CGA-effect, since $q = 1$ therefore we shall us the formulae (iv), (v) and (vi). Direct numerical application gives: $F_{\mathrm{D}} = 3.159948 \times 10^{26}\ \mathrm{N}$; $\Delta P = 1.975787 \times 10^{-1}\ \mathrm{s}$; $\Delta v = 4.302114\,\mathrm{m\,s^{-1}}$.

7.3. Double pulsar PSR J0737- 3039

The PSR J0737-3039 is the first double pulsar discovered in 2003 at Australia's Parkes Observatory by an international team led by the radio astronomer Marta Burgay during a high-latitude pulsar survey [34] which consists of two pulsars orbiting the common center of mass in a slightly eccentric orbit (e = 0.0877) of only 2.4-hr orbital duration and pulse period of 22.7 ms. It was immediately found to be a member of the most extreme binary system ever discovered [35]: its short orbital period is combined with a remarkably high value of the observed periastron advance ($\dot{\omega}_{\mathrm{OBS}} = 16.9\ \mathrm{deg/yr}$), *i.e*., four times larger than for PRS B1913+16! Like before, we will show that this double pulsar represents a truly unique gravitational laboratory for CGA by investigating the CGA-effects. According to the CGA, this is mainly due to the fact that the magnitude of the mutual dynamic gravitational force for the double pulsar PSR J0737-3039 is eight times larger than for PRS B1913+16 as we will see. We have according to [36] the following orbital and stellar parameters: $e = 0.0877$; $P = 0.102251\ \mathrm{d}$; $\dot{\omega}_{\mathrm{OBS}} = 16.9\ \mathrm{deg/yr}$; $a = 8.8 \times 10^{8}\ \mathrm{m}$; $M_A = 1.338 M_\Theta$; $M_B = 1.249 M_\Theta$; $q = 0.9334$. Since $e << 1/4$ and $q \approx 1$; therefore the eccentricity-mass ratio function (62), scalar parameter (63) and the formula (59) take, respectively, the form :

$$f(e,q) = 1; \quad M = M_A + M_B;$$

and

$$\dot{\omega}_{\mathrm{CGA}} = \frac{f(e,q)}{2}\left[\frac{G(M_A+M_B)P}{c_0\,a^2}\right]^2 .$$

-*Numerical Application*: $f(e,q)=1$; $G(M_A+M_B)P=3\times10^{24}\ \mathrm{m^3\,s^{-1}}$; $c_0\,a^2=2.3116\times10^{26}\ \mathrm{m^3\,s^{-1}}$. After substitution in the above formula, we get

$$\dot{\omega}_{\mathrm{CGA}} = 17.096440\ \mathrm{deg/yr}. \tag{77}$$

This is in good agreement with the observed value of $\dot{\omega}_{\mathrm{OBS}} = 16.9$ deg/yr. For the other CGA-effects, we have from the formulae (iv), (v) and (vi), for the case $q\approx 1$: $F_{\mathrm{D}} = 4.677426\times10^{27}$ N; $\Delta P = 1.174517\times10^{-1}$ s; $\Delta v = 16.632930\ \mathrm{m\,s^{-1}}$. As it was already mentioned, the magnitude (4.677426×10^{27} N) of the mutual dynamic gravitational force for PSR J0737-3039 is eight times larger than (5.831340×10^{26} N) for PRS B1913+16 that's why the high value of the CGA-apsidal motion rate (17.096440 deg/yr) is four times larger than (4.213832 deg/yr). Now, we can affirm from the study of the solar system, eclipsing binary star systems and binary pulsars that the CGA, as a gravity theory, is capable of predicting some old and new gravitational effects without evoking the curvature of space-time since the CGA is exclusively established in the framework of Euclidean geometry and Galilean relativity principle.

8. Conclusion

The CGA could be regarded as an alternative gravitational model to compare with the others that have already existed for a long time. As we have seen, the CGA enabled us to study and solve some old and new problems related to gravitational phenomena through a novel comprehension and interpretation of the *gravity* itself; the famous Newton's law of gravitation was corrected and reformulated in a new more general form.

References

[1] M.E. Hassani, Galilean Electrodynamics **20**, Special Issues 3, 54 (2009)
[2] M.E. Hassani, Galilean Electrodynamics **22**, Special Issues 3, 43 (2011)
[3] to be appeared in Galilean Electrodynamic

[4] C.T. Ridgely, Eur. J. Phys. **31**, 949 (2010)

[5] W. De Sitter, MNRAS **76**, 699, (1916)
[6] W. De Sitter MNRAS **77**, 155, (1916)
[7] W. De Sitter MNRAS **78**, 3, (1917)
[8] L. Silberstein, '*The Theory of Relativity*' 2nd edition; London: The Macmillan Company, Ltd., (1924)
[9] T. Levi-Cevita, Am. J. Math. **59**, 225 (1937)
[10] Z. Kopal, 'Close Binary Systems', Wiley, New York (1959)
[11] E.F. Guinan and F.P.Maloney, A J, **90**, 1519 (1985)}
[12] F.P. Maloney, E.F.Guinan and P.T.Boyd, AJ, **98**, (1989)
[13] J. Anderson, J.M. Garcia, A. Gimenez and B. Nordström, A&A, **174**, 107 (1987)
[14] J.F. Burns, E.F.Guinan and J.J.Marshall, IBVS **4363**, 1 (1996)
[15] C. H. S. Lacy, A. Claret and J. A. Sabby, A J, **128**, 134 (2004)
[16] A. Dariush, arXiv: astro-ph/0508414v3 (2005)
[17] J.W. Moffat, Phys. Rev. D, **19**, 3554 (1979)
[18] J.W. Moffat, ApJ, **287**, L77 (1984)
[19] J.W. Moffat, Phys. Rev. D, **39**, 474 (1989)
[20] R.W. Hilditch, Publ. Astron. Soc. Pacific, **84**, 519 (1972)

[21] R.W. Hilditch, Mem. Roy. Astron. Soc., **76**, 1, (1972)
[22] Kh. F. Khalliulin and V.S. Kozyreva, Ap&SS **94**, 115 (1983)
[23] E.F. Guinan, J.J. Marshall and F.P. Maloney, IBVS **4101**, 1 (1994)
[24] R.A. Rossiter, AJ, **60**, 15 (1924)
[25] D.B. McLaughlin, AJ, **60**, 15 (1924)
[26] S. Albrecht *et al*., Nature **461**, 373 (2009)
[27] C. H. S. Lacy, AJ, **115**, 801, (1998)
[28] C. H. S. Lacy, AJ, **105**, 1096, (1993)
[29] R.A. Hulse and J.H. Taylor, ApJ, **195**, L51 (1975)
[30] J.M. Weisberg and J.H.Taylor, in: F. A . Rasio and I. H. Stairs (Eds), Binary Radio Pulsars, ASP Conf. Series, Vol. 328, pp.25-32 (2005)
[31] A. Wolszczan, Nature, **350**, 688 (1990)
[32] I.H. Stairs *et al*., ApJ, **581,** 501 (2002)
[33] D.J.Nice, R.W. Sayer and J.H.Taylor, ApJ, **466**, L87 (1996)
[34] M. Burgay *et al*., Nature **426**, 531 (2003)
[35] A.G. Lyne *et al*., Science **303**, 1153 (2004)
[36] M. Kramer et al., in: P. Chen, E. Bloom, G. Madjeski, V. Patrosian. (Eds.), Proceedings of the 22nd Texas Symposium on Relativistic Astrophysics. (2005)