Logic Systems

Lattices,
classical logic and
quantum logic

Logic – Lattice structure

- A lattice is a set of elements $a, b, c,$ ...that is closed for the connections $\cap$ and $\cup$. These connections obey:
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  - The set is partially ordered. With each pair of elements a, b belongs an element c, such that $a \subset c$ and $b \subset c$.
  - The set is a $\cap$ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cap b$.
  - The set is a $\cup$ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cup b$.
  - The set is a lattice if it is both a $\cap$ half lattice and a $\cup$ half lattice.

Partially ordered set

- The following relations hold in a lattice:

$a \cap b = b \cap a$
$(a \cap b) \cap c$
$= a \cap (b \cap c)$
$a \cap (a \cup b) = a$
$a \cup b = b \cup a$
$(a \cup b) \cup c$
$= a \cup (b \cup c)$
$a \cup (a \cap b) = a$

- has a partial order inclusion $\subset$:

  $$a \subset b \Leftrightarrow a \subset b = a$$

- A complementary lattice contains two elements n and e with each element a an complementary element a'

  $a \cap a' = n \quad a \cap n = n$
  $a \cap e = a \quad a \cup a' = e$
  $a \cup e = e \quad a \cup n = a$

Orthocomplemented lattice

- Contains with each element a an element a'' such that:

$$a \cup a'' = e$$
$$a \cap a'' = n$$
$$(a'')'' = a$$
$$a \subset b \Leftrightarrow b'' \subset a''$$

Distributive lattice

$$a \cap (b \cup c)$$
$$= (a \cap b) \cup (a \cap c)$$
$$a \cup (b \cap c)$$
$$= (a \cup b) \cap (a \cup c)$$

Modular lattice

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c))$$

Classical logic is an orthocomplemented modular lattice

Weak modular lattice

- There exists an element d such that

$$a \subset c \Leftrightarrow (a \cup b) \cap c = a \cup (b \cap c) \cup (d \cap c)$$

- where d obeys:

$$(a \cup b) \cap d = d$$

$$a \cap d = n \qquad b \cap d = n$$

$$[(a \subset g) \text{ and } (b \subset g) \Leftrightarrow d \subset g$$

Atoms

- In an atomic lattice

$$\exists_{p \in L} \forall_{x \in L} \{x \subset p \Rightarrow x = n\}$$

$$\forall_{a \in L} \forall_{x \in L} \{(a < x < a \cap p)$$

$$\Rightarrow (x = a \; or \; x = a \cap p)\}$$

p is an atom

Logics

- Classical logic has the structure of an orthocomplemented distributive modular and atomic lattice.
- Quantum logic has the structure of an orthocomplented weakly modular and atomic lattice.
- Also called orthomodular lattice.

Hilbert space

- The set of closed subspaces of an infinite dimensional separable Hilbert space forms an orthomodular lattice
- Is lattice isomorphic to quantum logic

Back

Hilbert logic

- Add linear propositions
  - Linear combinations of atomic propositions
- Add relational coupling measure
  - Equivalent to inner product of Hilbert space
- Close subsets with respect to realational coupling measure

- Propositions ⇔ subspaces
- Linear propositions ⇔ Hilbert vectors

Superposition principle

Linear combinations of linear propositions are again linear propositions that belong to the same Hilbert logic system

Isomorphism

- Lattice isomorhic
  - Propositions ⇔ closed subspaces
- Topological isomorphic
  - Linear atoms ⇔ Hilbert base vectors

The Higgs mechanism

The HBM has its own solution

http://www.youtube.com/watch?v=JqNg819PiZY

Higgs mechanism

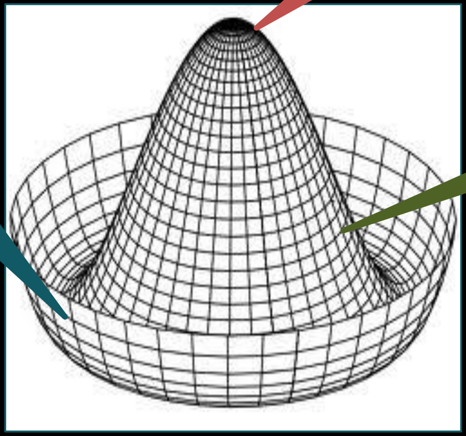
Complex number based
QP

Energy V(ψ)

Goldstone boson
at bottom

Higgs boson
at wall

Wave function



$$\psi(\rho,\theta) = \rho(f)\, e^{i\theta}$$

$$\mathcal{L} = \partial_\mu\psi\partial_\mu\psi^*$$

Lagrangian

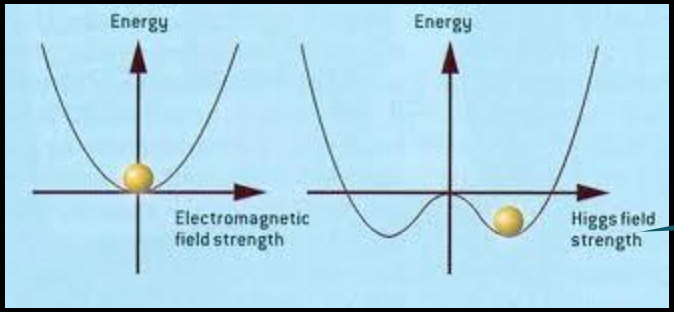
$$\partial_\mu\psi = (\partial_\mu\rho + i\rho\partial_\mu\theta)e^{i\theta}$$

$\longrightarrow \psi$

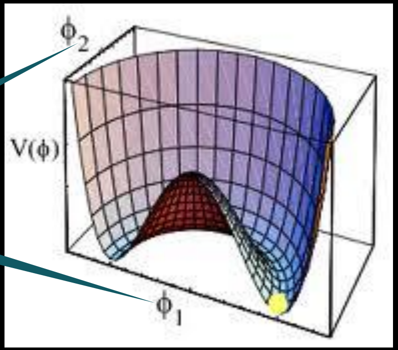
$$\mathcal{L} = (\partial_\mu\rho)^2 + i\partial_\mu\theta\,[\rho\partial_\mu\rho^* - \rho^*\partial_\mu\rho] + \rho^2(\partial_\mu\theta)^2$$

Thus, $e^{i\theta}$ is no symmetry!

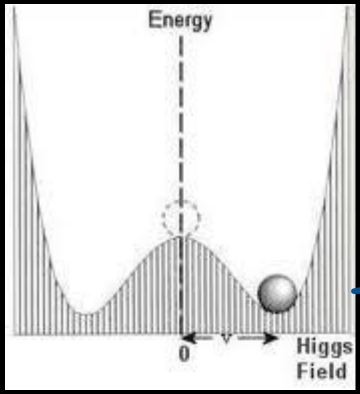
Higgs mechanism⇔HBM



Two fields



Qpattern pair



$$\nabla\psi^{x} = m\,\psi^{y}$$

Coupling equation

Mexican hat fields graph

Particle oscillates between fields at lowest energy

Gauge transformation

Covariant derivative

$$D_\mu\psi = \partial_\mu\psi - i\,A_\mu\psi$$

$$= (\partial_\mu\theta + A_\mu)i\rho e^{i\theta}$$

$$A'_\mu = \partial_\mu\theta + A_\mu$$

The new Lagrangian is

$$\mathcal{L} = D_\mu\psi D_\mu\psi^* = f^2(\partial_\mu\theta + A_\mu)^2 = f^2{A'_\mu}^2$$

θ is replaced by a new field A'_μ

The factor f represents mass