

The general Compton effect

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ABSTRACT

In the special relativity theory and the quantum theory, the Compton effect is the effect about the standing electron and the moving photon. The general Compton effect is the case that the moving photon impulses the moving electron. In this time, the photon's sense is the opposite sense of the moving electron.

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I. Introduction

This article treats that the general Compton effect.

The Compton effect is the case that the moving photon impulses the standing electron.

The energy conservation is

$$h\nu + m_0c^2 = h\nu' + m_0\gamma c^2 \quad (1), \quad \gamma = 1/\sqrt{v_{f0}^2/c^2}$$

The momentum conservation is

$$\frac{h}{\lambda} = m_0\gamma\beta c \cos\phi + \left(\frac{h}{\lambda'}\right) \cos\theta \quad (2)$$

$$0 = -m_0\gamma\beta c \sin\phi + \left(\frac{h}{\lambda'}\right) \sin\theta \quad (3)$$

$$\gamma = 1/\sqrt{v_{f0}^2/c^2}, \quad \beta = v_{f0}/c$$

v_{f0} is the electron's velocity after the moving photon impulses the standing electron.

If use Eq(1),Eq(2),Eq(3),

$$\lambda' - \lambda = \frac{h}{m_0c} (1 - \cos\theta) \quad (4), \quad \text{Compton wavelength } \lambda_C = \frac{h}{m_0c}$$

II. Additional chapter

The general Compton effect is the case that the moving photon impulses the moving electron. In this time,

if the photon's sense is the opposite sense of the moving electron, the general Compton effect is

The energy conservation is

$$h\nu + \gamma m_0c^2 = h\nu' + m_0\gamma' c^2 \quad (5), \quad \gamma = 1/\sqrt{v_{i0}^2/c^2}, \gamma' = 1/\sqrt{v_{f0}^2/c^2}$$

The momentum conservation is

$$\frac{h}{\lambda} - m_0\gamma\beta c = m_0\gamma' \beta' c \cos\phi + \left(\frac{h}{\lambda'}\right) \cos\theta \quad (6)$$

$$0 = -m_0\gamma' \beta' c \sin\phi + \left(\frac{h}{\lambda'}\right) \sin\theta \quad (7)$$

$$\gamma = 1/\sqrt{v_{i0}^2/c^2}, \gamma' = 1/\sqrt{v_{f0}^2/c^2}, \quad \beta = v_{i0}/c, \beta' = v_{f0}/c$$

v_{i0} is the moving electron's velocity before the moving photon impulses the moving electron.

v_{f0} is the moving electron's velocity after the moving photon impulses the moving electron.

Eq(6),Eq(7) is

$$\left[\frac{h}{\lambda} - m_0\gamma\beta c - \left(\frac{h}{\lambda'}\right)\cos\theta\right]^2 = m_0^2\gamma'^2\beta'^2c^2\cos^2\phi \quad (8)$$

$$\left(\frac{h}{\lambda'}\right)^2\sin^2\theta = m_0^2\gamma'^2\beta'^2c^2\sin^2\phi \quad (9)$$

Therefore,

$$\begin{aligned} \left(\frac{h}{\lambda}\right)^2 + m_0^2\gamma^2\beta^2c^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\frac{h}{\lambda}m_0\gamma\beta c + 2\frac{h}{\lambda'}\cos\theta m_0\gamma\beta c - 2\frac{h^2}{\lambda\lambda'}\cos\theta \\ = m_0^2\gamma'^2\beta'^2c^2 \quad (10) \end{aligned}$$

Eq(5) is

$$\frac{h}{\lambda} - \frac{h}{\lambda'} = m_0c(\gamma' - \gamma) \quad (11)$$

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 = \left(\frac{h}{\lambda}\right)^2 - 2\frac{h^2}{\lambda\lambda'} + \left(\frac{h}{\lambda'}\right)^2 = m_0^2c^2(\gamma'^2 - 2\gamma'\gamma + \gamma^2) \quad (12)$$

If Eq(10) minus Eq(12),

$$\begin{aligned} 2\frac{h^2}{\lambda\lambda'}(1 - \cos\theta) - 2\frac{h}{\lambda\lambda'}m_0\gamma\beta c(\lambda' - \lambda\cos\theta) \\ = -m_0^2\gamma'^2c^2(1 - \beta'^2) - m_0^2c^2\gamma^2(1 + \beta^2) + 2m_0^2c^2\gamma\gamma' \\ = -m_0^2c^2 - m_0^2c^2\gamma^2(1 + \beta^2) + 2m_0^2c^2\gamma\gamma' = 2m_0^2c^2\gamma(\gamma' - \gamma) \quad (13) \end{aligned}$$

Therefore,

$$\lambda\lambda' = [h^2(1 - \cos\theta) - hm_0\gamma\beta c(\lambda' - \lambda\cos\theta)]/[m_0^2c^2\gamma(\gamma' - \gamma)] \quad (14)$$

Eq(11) is

$$\frac{h}{\lambda} - \frac{h}{\lambda'} = \frac{h}{\lambda\lambda'}(\lambda' - \lambda) = m_0c(\gamma' - \gamma) \quad (15)$$

III. Conclusion

Therefore, if Eq(14) insert Eq(15), the general Compton effect, the case that the moving photon impulses the moving electron is

$$\lambda' - \lambda = \frac{h(1 - \cos\theta)}{m_0c\gamma} - \beta(\lambda' - \lambda\cos\theta) \quad (16)$$

$$\lambda' = \frac{1}{1 + \beta} \left[\frac{h}{m_0c\gamma} (1 - \cos\theta) + \lambda(1 + \beta\cos\theta) \right] \quad (17)$$

$$\begin{aligned}\lambda' - \lambda &= \frac{1}{1 + \beta} \left[\frac{h}{m_0 c \gamma} (1 - \cos \theta) + \lambda(1 + \beta \cos \theta) - (1 + \beta)\lambda \right] \\ &= \frac{1}{1 + \beta} \left[\frac{h}{m_0 c \gamma} - \lambda \beta \right] (1 - \cos \theta) \quad (18)\end{aligned}$$

$$\gamma = 1 / \sqrt{v_{i0}^2 / c^2}, \beta = v_{i0} / c$$

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