Metaphysics of the free Fock space with local and global information

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Abstract

A new interpretation of the basic vector of the free Fock space (FFS) and the FFS is proposed. The approximations to various equations with additional parameters, for n-point information (n-pi), are also considered in the case of non-polynomial nonlinearities.

Key words: basic, generating and state vectors, local and global, Cuntz relations, perturbation and closure principles, homotopy analysis method, Axiom of Choice, consilience.

Contents

1 Introduction and basic axioms

It can sometimes happen that scientists resemble somewhat the Moliere's hero, Mr Jourdain, who did not know that he was speaking prose. I see this analogy in fact in ignoring of the **free Fock space** (FFS) as an arena in which description should be made of various complex phenomena and processes like turbulence, economic or general relativity quantum processes. My faith in possibilities of the FFS comes from the fact that in this space you can describe both macro- and micro-physics, see [1]. Moreover, in these both cases one can introduce vacuum vectors as well as the cration and annihilation operators and develope similar calculus, [2, 1],[4]. It turns out that in the FFS all these complex phenomena can be formulated in a linear way. Moreover, in the FFS in a natural way one side-invertible operators appear by means of which such important concepts as injectivity and surjectivity can be defined in a way that does not depend on elements of a set. This, in some sense, suggests a connection of the FFS with the category theory - promising land of combination of different concepts, see [12],[13] and Appendix2.

The vector $|V>$ describing, e.g., a classical or quantum system can be expanded in an infinite series of the linearly independent orthonormal vectors $|\tilde{x}_{(n)}>$:

$$|V>=\sum_{n=1}\int d\tilde{x}_{(n)}V(\tilde{x}_{(n)})|\tilde{x}_{(n)}> +V(0)|0> \tag{1}$$

The vectors (1) form the **Fock space** when basic vectors $|\tilde{x}_{(n)}>$ are constructed in a certain way with the help of creation operators and the basic vector |0>, see below. The adjective '**free**' **before** '**Fock space**' comes from the fact that the 'components' $V(\tilde{x}_{(n)})$ of vector |V> are not restricted by any symmetries, such as permutation invariance. Algebraic realization of this fact is given by creation and annihilation operators satisfying the Cuntza relations, see below. See also [5]. In our opinion, the **algebraic thread introduced by the FFS** is more fruitful and essentional than approaches based on orthonormal vectors (1) or equivalent approach based on the linear functionals:

$$V = \sum_{n=1} \int d\tilde{x}_{(n)} V(\tilde{x}_{(n)}) \rho(\tilde{x}_{(n)})$$

with the 'arbitrary' functions $\rho(\tilde{x}_{(n)}) \equiv \rho(\tilde{x}_1, ..., \tilde{x}_n)$ and the corresponding functional derivatives: $\delta/\delta\rho(\tilde{x}_1, ..., \tilde{x}_n)$, [11]. Of course, all these representations are equivalent.

The components of the vectors $\tilde{x}$, on which the components $V(\tilde{x}_{(n)})$ of the vector |V> and the basic vectors $|\tilde{x}_{(n)}>$depend, contain the time-space and other (discrete) variables. Moreover, for the most part the work, we would assume that the time-space coordinates are discrete. However, for greater transparency in the work, we use the symbol $\int$ instead of $\sum$.

The adjective 'free' also refers to the fact that the vector variables $\tilde{x}_{(n)} = \tilde{x}_1, ..., \tilde{x}_n$are independent of each other, for example, their time components are not equal to one another. In other words, under the sign of smoothing or averaging, $<>$, we mean rather the tensor product of fields so that the *multi-time* n-pi are considered. In this way the concept of the **state vector of the system** is substituted by the concept of the **generating vector of the system**. It seems that this replacement is particularly useful in the areas, where the averagings or smoothings are used, see also [4, 1], [3].

Interpretation of |V>

The 'components' $V(\tilde{x}_{(n)})$ have physical interpretations, e.g., they can be some summarizing, averaging or smoothing of product of the field φ: $<\varphi(\tilde{x}_1)\cdots\varphi(\tilde{x}_n)>$, see e.g., [3], [1], or App.1and App.4. They are called the ***n-point (quasilocal) information (about the system or object), (n-pi)***.

The vector $|\tilde{x}_{(n)}>$ standing at the given n-pi represents rather everything that leads to the determination (measurement) of the n-pi $V(\tilde{x}_{(n)})$. The vector |V> thus refers to a physical system, such as electromagnetic fields or the economic system described by an infinite system of n-pi and associated to them infinite system of measuring arrangements. Assuming the possibility of perfect isolation from the rest of the universe, we can assume that anything beyond the given system and used instrumentation does not exist. Since the vector |0> does not represent any measuring instrument, and the factor V (0) does not include any local information on the system, we can assume that $V(0)|0> \equiv |0>_{info}$ represents a local vacuum. At this point, we break the tyranny of the philosophy of eigenvectors and eigenvalues of at least some unifying scientific descriptions. See also [23].

We accept the following assumptions (axioms) concerning certain vectors belonging to the FFS **formed with vectors related to particular or total information about the Universe**:

Axioms:

ASSUMPTION1 (A1):

There is a vector $|U>$ representing the Universe (Total system,Total (Global) information)

ASSUMTION2 (A2):

Vectors |V> like (1) represent systems belonging to the Universe (subsystems, (quasi) global information)

ASSUMPTION3 (A3):

There is a vector $|0>_{info}$ representing the local vacuum (no local information) such that:

$$|0>_{info} \sim |0> \sim |U> \tag{2}$$

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Comments:

The Universe is understood in the sense of category theory, see [13] and Appendix2.

The vector $|0>$ is a basic vector of the FFS by means of which the orthonormal base vectors $|\tilde{x}_{(n)}>$, for $n = 0, 1, 2, ..., \infty$, and dual to them $<\tilde{x}_{(m)}|$, for $m = 0, 1, 2, ..., \infty$, are constructed. We have

$$V(\tilde{x}_{(n)}) = <\tilde{x}_{(n)}|V> \tag{3}$$

for n=1,2,.... and, for n=0, $V(0) = <0|V>$.

In the FFS the base vectors $|\tilde{x}_{(n)}>$ are constructed using the vector $|0>$ and the creation operators $\hat{\eta}^{\star}(\tilde{x})$ as follows:

$$|\tilde{x}_{(n)}> = \hat{\eta}^{\star}(\tilde{x}_1) \cdots \hat{\eta}^{\star}(\tilde{x}_n)|0> \tag{4}$$

In other words, every measuring arrangements are 'constructed' from the basic vector |0>. This probably means that in A3 you can identify the vector |0> with the vector |U>.

$$|U> = |0> \tag{5}$$

Moreover, proportional identification of these two vectors with a local vacuum $|0>_{info}$ expresses the possibility of isolation of the given system from the rest of the Universe. The rest of the Universe identified with the whole Universe works like a vacuum system.

This is consistent with the intuitive understanding of the concepts - local and global. Moreover, as we know, in various areas of material and spiritual world, global phenomena have negligible or no impact on local phenomena, and vice vers, and it seems surprisingly that this property is confirmed by the derived equations.

To better feel the interaction, or rather the lack of it, between global and local phenomena in economics, see [10].

Cuntz relations

In algebraic language the FFS can be constructed by means of creation and annihilation operators satisfying the Cuntz relations:

$$\hat{\eta}(\tilde{y})\hat{\eta}^{\star}(\tilde{x}) = \delta(\tilde{y} - \tilde{x}) \cdot \hat{I} \tag{6}$$

and

$$\hat{\eta}(\tilde{y})|0> = 0, \quad < 0|\hat{\eta}^{\star}(\tilde{x}) = 0 \tag{7}$$

where all variables $\tilde{x}, \tilde{y}$ are discrete quantities and $\hat{I}$ is the unit operator in the FFS. In this way δ denotes rather Kronecker not Dirac symbol. In a sense, the Cuntz relations remind us of the relations: $\partial/\partial x_i \cdot x_j = \delta_{ij}$, but from the Cuntz relations the analogue of the Leibniz identity does not result, see [4]. For simplicity, we assume that the creation operators $\hat{\eta}^{\star}(\tilde{x})$ are Hermitian conjugate to the appropriate annihilation operators $\hat{\eta}(\tilde{x})$.

Additional remarks

The components $V(\tilde{x}_{(n)})$; $n = 1, 2, ..., \infty$ of the vector $|V>$, standing at the vectors $|\tilde{x}_{(n)}>$ in the expansion (1) quantitatively and specifically describe separate parts of the system at varies times, see, e.g., [1]. In fact, we consider here the multitime n-pi and in fact this is responsible that such vectors as $|V>, |\tilde{x}_{(n)}>$ do not represent states!

See Final remarks in order to clarify the content of the work!

Illustration of axioms:

$$\begin{array}{cc} \longrightarrow\longrightarrow\longrightarrow\longrightarrow\longrightarrow\rightarrow & A1 \\ \longrightarrow\longrightarrow\longrightarrow & A2 \\ \leftrightarrow & A3 \end{array}$$

2 Equations for the generating vector |V>

The vector $|V>\in FFS$ can describe an oscillator or the Universe in a depence on the choice of the linear operator $\hat{A}$ appearing in the equation:

$$\hat{A}|V> = |0>_{info} \tag{8}$$

In the both cases, the *vacuum vector* $|0>_{info}\sim\ |0>$. Eq.8 restricting n-pi generated by the vector $|V>$ was derived in previous papers [1, 4], so we only here remind you some its properties, for a more, see App1. It turns out that the operator $\hat{A}$ can be decompose in three parts each of which is related to a specifice interaction of the system and we get the following equation:

$$\left(\hat{L}+\lambda\hat{N}+\hat{G}\right)|V>=\hat{P}_0\hat{L}|V>+\lambda\hat{P}_0\hat{N}|V>+\hat{P}_0\hat{G}|V>\equiv|0>_{info} \qquad (9)$$

where the vectot $|0>_{info}$ describes so called the local information vacuum. In other words, no local information is included in this vector. In this paper the vector $|0>_{info}$is called the *classical vacuum.*

The operator $\hat{L}$ is a right invertible operator, which is diagonal:

$$\hat{P}_n\hat{L}=\hat{L}\hat{P}_n \qquad (10)$$

with respect to the projectors $\hat{P}_n$, where the project $\hat{P}_n$projects on the n-th term in the expansion (1).

The operator $\hat{G}$ is a left invertible operator, which is lower triangular operator:

$$\hat{P}_n\hat{G}=\sum_{m<n}\hat{P}_n\hat{G}\hat{P}_m \qquad (11)$$

and the operator $\hat{N}$ can be right or left invertible operator, see [4] (all depend on a definition of the operator-valued functions). In the case of polynomial operators this is an upper triangular operator:

$$\hat{P}_n\hat{N}=\sum_{n<m}\hat{P}_n\hat{N}\hat{P}_m \qquad (12)$$

see [1].

All these operators are linear in the FFS.

The diagonal operator $\hat{L}$ is related to the linear interaction among constituents of the system and, or, describes the kinematic part of theory usually responsible for additional conditions needed for the unique solutions. In the case of quantum field theory this part of the operator $\hat{A}$ corresponds to lack of interaction between the system components. If we take into account that $\hat{L}$ is a surjection (the image of the domain of a surjective function completely covers the function's codomain) then it is surprising that **Axiom of Choice** of set theories is equivalent to saying: **any surjective function has a right inverse!** See App.2.

The operator $\hat{N}$ is related to the nonlinear interaction of the constituents including self-interaction as well. It is this property which leads, independently of right or left invertability of the operator $\hat{N}$, to an infinite system of equations to n-pi, see [4, 1].

The last, lower trangular operator $\hat{G}$ expresses very deep, qualitative properties of described systems like this that the system is immersed in a given external field, or that the system is subjecting to certain constraints (commutation relations) that are implemented without the participation of reaction forces, see ([1]; Secs 3 and 4.2). In the case of **General Relativity** the operator $\hat{G}$ is related to the stress-energy momentum tensors and in this case should be rather denoted as

$$\hat{G} \equiv \hat{T}$$

In the case of **Macroscopic Gravity** (MG) to descibe large scale structures entering the Cosmos, the above operator can be treated as a random quantity from which the n-pi (e.g.,correlation functions) depend parametrically. After solving in formal way the equations for the n-pi obtained by means of averaging with respect to initial and boundary conditions, for Eq.71, we can calculate appropriate expectation values with respect to $\hat{T}$. In this way the averaging problem related to the space-time averaging can be avoided, see [22] and App.6.

Taking into account that

$$\hat{P}_0 \hat{G} = 0 \tag{13}$$

we see that Eq.9, for projection $\hat{P}_0$, is identically satisfied. However, **we have introduced such identity to have in its result, in the l.h.s. of (9), where it is possible, the right invertible operators**, see [1, 4]. The term, which have had appeared in the r.h.s., we interpreted as a (local) vacuum. In the present study we want to examine its role.

3 The role of the classical vacuum represented by the vector $|0>_{info}$

It turns out that complement of the equations for the generating vector $|V>$ in such a way that the operators appearing in them were at least one side invertible, caused the appearance in homogeneous equations the vector $|0>_{info}$, which we called the (local) vacuum.

3.1 $\hat{L}$ is a right invertible operator. The perturbation principle

Taking into account that the operator $\hat{L}$ is right invertible it means that a right inverse exists $\hat{L}_R^{-1}$, for which

$$\hat{L}\hat{L}_R^{-1} = \hat{I} \tag{14}$$

With the help of operator $\hat{L}_R^{-1}$ we can rewrite Eq.9 as follows

$$\left(\hat{I} + \lambda \hat{L}_R^{-1} \hat{N} + \hat{L}_R^{-1} \hat{G}\right) |V> = \hat{L}_R^{-1} |0>_{info} + \hat{P}_L |V> \tag{15}$$

where a projector

$$\hat{P}_L = \hat{I} - \hat{L}_R^{-1}\hat{L} \tag{16}$$

projects on the null space of the operator $\hat{L}$, see [9]. It is a diagonal projector, see (10), and its choice, among other things, depends on a choice of a right inverse operator $\hat{L}_R^{-1}$.

We will assume that solutions are symmetric:

$$|V >= \hat{S}|V > \tag{17}$$

for example - the permutation symmetric. Because the operator $\hat{L}_R^{-1}\hat{G}$ is a lower triangular operator and the operator $\hat{S}$ is diaginal, see [5] , the Eq.15 can be equivalently transformed further as follows:

$$\left(\hat{I} + \lambda \left(\hat{I} + \hat{S}\hat{L}_R^{-1}\hat{G}\right)^{-1} \hat{S}\hat{L}_R^{-1}\hat{N}\right)|V >=$$
$$\left(\hat{I} + \hat{S}\hat{L}_R^{-1}\hat{G}\right)^{-1} \left(\hat{S}\hat{L}_R^{-1}|0 >_{info} + \hat{S}\hat{P}_L|V >\right) \tag{18}$$

This equation allows the next (formal) step:

$$|V >= \left(\hat{I} + \lambda \left(\hat{I} + \hat{S}\hat{L}_R^{-1}\hat{G}\right)^{-1} \hat{S}\hat{L}_R^{-1}\hat{N}\right) \bullet^{-1}$$
$$\left(\hat{I} + \hat{S}\hat{L}_R^{-1}\hat{G}\right)^{-1} \left(\hat{S}\hat{L}_R^{-1}|0 >_{info} + \hat{S}\hat{P}_L|V >\right) \tag{19}$$

which can be used for further calculations. Note first that the expression $\left(\hat{S}\hat{L}_R^{-1}|0 >_{info} + \hat{S}\hat{P}_L|V >\right)$ is not yet known. The vector $\hat{S}\hat{P}_L|V >$, arbitrary from the standpoint of Eq.9, we fix by taking into account the *perturbation principle* according to which

$$\hat{S}\hat{P}_L|V >= \hat{S}\hat{P}_L|V >^{(0)} \tag{20}$$

where superscript (0) means that $\lambda = 0$ in the l.h.s. of Eq.17, see, however, App.7.

We still do not know the vacuum vector $|0 >_{info}$, see Eq.9. We assume the following *normalization condition*:

$$\hat{P}_0\hat{L}|V >= |0 > \tag{21}$$

and from Eq.16 we have upon the term $\hat{P}_0\hat{N}|V >$the following equation:

$$\hat{P}_0\hat{N}|V >= \hat{P}_0\hat{N} \bullet (r.h.s.)\ of\ (19)) \tag{22}$$

or equivalently:

$$< 0|\hat{N}|V >=< 0|\hat{N} \bullet (r.h.s.)\ of\ (19) \tag{23}$$

This is the equation for the element $< 0|\hat{N}|V >$, which also occurs in the r.h.s. of Eq.19, see Eq.9.

Denoting bra-vector $< 0|\hat{N} \equiv < \Psi|$ we postulate the following equation on $< \Psi|$:

$$\frac{\delta|V>}{\delta < \Psi|} = 0 \tag{24}$$

This equation comes from the fact that the extension of the operator $\hat{N}$ about the element $\hat{P}_0\hat{N}$ is in a sense, arbitrary. Hence the desire to minimalize with respect to changes of $< \Psi|$ computed quantities .

3.2 A theory with two parameters (coupling constants) and left invertible $\hat{N}(\lambda_2)$

In [2], see also [3], we considered a theory with two parameters λ_1 and $\lambda_2$with the following equation on the generating vector |V>:

$$\begin{aligned}&\left(\hat{L} + \lambda_1\hat{N}(\lambda_2) + \hat{G}\right)|V >= \\ &\hat{P}_0\left(\hat{L} + \lambda_1\hat{N}(\lambda_2) + \hat{G}\right)|V >\equiv |0 >_{info}\end{aligned} \tag{25}$$

In this case the parameter $\lambda_1$is called the *major coupling constant*. We additionally assume that the operator $\hat{N}$ is a left invertible, see (44). It means that an operator $\hat{N}_l^{-1}(\lambda_2)$ exists, for which

$$\hat{N}_l^{-1}(\lambda_2)\hat{N}(\lambda_2) = \hat{I} \tag{26}$$

see [4] and Eq.47in the present paper. Hence and from Eq.25 we get

$$\left(\hat{N}_l^{-1}(\lambda_2)(\hat{L} + \hat{G}) + \lambda_1\hat{I}\right)|V >= \hat{N}_l^{-1}(\lambda_2)|0 >_{info}= 0 \tag{27}$$

Taking into account that the operator $\hat{N}_l^{-1}(\lambda_2)(\hat{L}+\hat{G})$ is a right invertible with operator $(\hat{L} + \hat{G})_R^{-1}\hat{N}(\lambda_2)$ as its right inverse, we can equivalently rewrite the above equation as

$$\left(\hat{I} + \lambda_1(\hat{L} + \hat{G})_R^{-1}\hat{N}(\lambda_2)\right)|V >= \hat{\Pi}(\lambda_2)|V > \tag{28}$$

with projector

$$\begin{aligned}\hat{\Pi}(\lambda_2) = \quad &\hat{I} - (\hat{L} + \hat{G})_R^{-1}\hat{N}(\lambda_2)\hat{N}_l^{-1}(\lambda_2)(\hat{L} + \hat{G}) \equiv \\ &\hat{I} - (\hat{L} + \hat{G})_R^{-1}\hat{Q}_l(\lambda_2)(\hat{L} + \hat{G})\end{aligned} \tag{29}$$

In the last equation we have introduced the projector $\hat{Q}_l(\lambda_2) = \hat{N}(\lambda_2)\hat{N}_l^{-1}(\lambda_2)$. It is interesting to notice that similar equation as (28) we get if we multiply Eq.25 by a right inverse operator $(\hat{L} + \hat{G})_R^{-1}$:

$$\left(\hat{I}+\lambda_1(\hat{L}+\hat{G})_R^{-1}\hat{N}(\lambda_2)\right)|V>=$$
$$\hat{P}_{L+G}|V>+(\hat{L}+\hat{G})_R^{-1}|0>_{info} \tag{30}$$

with projector

$$\hat{P}_{L+G}=\hat{I}-(\hat{L}+\hat{G})_R^{-1}(\hat{L}+\hat{G}) \tag{31}$$

projecting on the null space of the operator $(\hat{L}+\hat{G})$. Comparing equations (28) with (30) we get the following equality upon undefined elements of the both equations:

$$\hat{\Pi}(\lambda_2)|V>=\hat{P}_{L+G}|V>+(\hat{L}+\hat{G})_R^{-1}|0>_{info} \tag{32}$$

3.3 A three parameter theory and one-side invertible operators $\hat{N}(\lambda_2)$

We consider the operator

$$\hat{N}=\frac{\hat{N}(\lambda_2)}{\hat{I}+\lambda_3\hat{N}(\lambda_2)} \tag{33}$$

We now have an equation:

$$\left(\hat{L}+\lambda_1\tfrac{\hat{N}(\lambda_2)}{\hat{I}+\lambda_3\hat{N}(\lambda_2)}+\hat{G}\right)|V>=$$
$$\hat{P}_0\left(\hat{L}+\lambda_1\tfrac{\hat{N}(\lambda_2)}{\hat{I}+\lambda_3\hat{N}(\lambda_2)}+\hat{G}\right)|V>\equiv|0>_{info} \tag{34}$$

with three parameters: the major coupling constant - λ_1, the minor coupling constant - λ_2 and the regularization parameter - λ_3, for which, if $\lambda_3=0$, then $\hat{N}=\hat{N}(\lambda_2)$. For the possible interpretation of these and other constants occurring in the operators $\hat{L},\hat{N}$ and $\hat{G}$ see App.5.

From Eq.34

$$\left\{\left(\hat{I}+\lambda_3\hat{N}(\lambda_2)\right)\left(\hat{L}+\hat{G}\right)+\lambda_1\hat{N}(\lambda_2)\right\}|V>=$$
$$\left(\hat{I}+\lambda_3\hat{N}(\lambda_2)\right)|0>_{info} \tag{35}$$

Left invertible operator $\hat{N}(\lambda_2)$

For a left invertible operator $\hat{N}(\lambda_2)$, see (35), we can transform this equation as follows:

$$\left\{\left(\hat{N}_l^{-1}(\lambda_2)+\lambda_3\hat{I}\right)\left(\hat{L}+\hat{G}\right)+\lambda_1\hat{I}\right\}|V>=$$
$$\left(\hat{N}_l^{-1}(\lambda_2)+\lambda_3\hat{I}\right)|0>_{info} \tag{36}$$

For a large absolute value of the major coupling constant λ_1, this equation can be solved by the perturbation method with the perturbation operator $\lambda_1^{-1}\hat{N}_l^{-1}(\lambda_2)\left(\hat{L}+\hat{G}\right)$. In this case, the zeroth order approximation is approximately described by the equation:

$$\left\{\frac{\lambda_3}{\lambda_1}\left(\hat{L}+\hat{G}\right)+\hat{I}\right\}|V>=\frac{\lambda_3}{\lambda_1}|0>_{info} \tag{37}$$

where we have assumed that $\lambda_1 \approx \lambda_3$. For $\lambda_1 >>> \lambda_3$,

$$|V>\approx\frac{\lambda_3}{\lambda_1}|0>_{info} \tag{38}$$

In other words, in this case, in the zeroth order approximation, no local information is contained in the generating vector |V>.

Right inverible operator $\hat{N}(\lambda_2)$

In this case there is an operator $\hat{N}_R^{-1}(\lambda_2)$ for which

$$\hat{N}(\lambda_2)\hat{N}_R^{-1}(\lambda_2)=\hat{I} \tag{39}$$

see Eq.35, and Eq.34 can be transformed as follows:

$$\left\{\hat{I}+\left[\lambda_3\hat{N}(\lambda_2)\left(\hat{L}+\hat{G}\right)\right]_R^{-1}\left(\hat{L}+\hat{G}+\lambda_1\hat{N}(\lambda_2)\right)\right\}|V>=$$
$$\hat{P}_N(\lambda_2)|V>+\left[\lambda_3\hat{N}(\lambda_2)\left(\hat{L}+\hat{G}\right)\right]_R^{-1}|0>_{info} \tag{40}$$

with the projector

$$\begin{aligned}\hat{P}_N(\lambda_2) &= \hat{I}-\left[N(\lambda_2)\left(\hat{L}+\hat{G}\right)\right]_R^{-1}\hat{N}(\lambda_2)\left(\hat{L}+\hat{G}\right)\equiv \\ \hat{I}- &\quad \left(\hat{L}+\hat{G}\right)_R^{-1}\hat{Q}_N(\lambda_2)\left(\hat{L}+\hat{G}\right)\end{aligned} \tag{41}$$

where the projector $\hat{Q}_N(\lambda_2)=\hat{N}(\lambda_2)_R^{-1}\hat{N}(\lambda_2)$. It turns out that for many polynomial operators $\hat{N}$, Eq.40 are closed, see [2, 5].

4 A few examples of right and left invertible operators

We give here a few examples of operators $\hat{N}$ which are right or left invertible operators and to which it is easy to construct appropriate inverses. Moreover, such operators appear in varies field theories. So, let us define

$$\hat{N} = \int d\tilde{x}\hat{\eta}^\star(\tilde{x})\hat{\eta}^2(\tilde{x}) + \hat{P}_0 \int d\tilde{x}\hat{\eta}(\tilde{x})f(\tilde{x}) \tag{42}$$

A right inverse to this operator is

$$\hat{N}_R^{-1} = 1/2 \int d\tilde{y}\hat{\eta}^\star(\tilde{y})^2\hat{\eta}(\tilde{y}) + 1/2 \int d\tilde{y}\hat{\eta}^\star(\tilde{y}) \tag{43}$$

if $\int d\tilde{x}f(\tilde{x}) = 2$. An example of **left invertible** operator is

$$\hat{N} \equiv \hat{N}(\lambda_2) = \int d\tilde{x}\hat{\eta}^\star(\tilde{x})H(\tilde{x}) \left(M[\tilde{x};\hat{\eta};\lambda_2]\right)_R^{-1} \hat{I} \tag{44}$$

where $(M[\tilde{x};\hat{\eta};\lambda_2])_R^{-1}$, for every $\tilde{x}$ is a right, or both sides, inverse to the operator $M[\tilde{x};\hat{\eta};\lambda_2]$ with a general operator-valued function M. As a simple example of operator-valued function, let us take

$$M[\tilde{x};\hat{\eta};\lambda_2] = \hat{I} - \lambda_2\hat{\eta}(\tilde{x}) \tag{45}$$

In the case of the right invertible operator (45) we have a large selection of right inverses, see [9], [2]. Our choice can be inspired by the formal formula:

$$\left(\hat{I} - \lambda_2\hat{\eta}(\tilde{x})\right)^{-1} = \hat{I} + \sum_{n=1}^{\infty} (\lambda_2\hat{\eta}(\tilde{x}))^n = \left(\hat{I} - \lambda_2\hat{\eta}(\tilde{x})\right)_R^{-1} \tag{46}$$

which, for a small absolute value of the minor coupling constant λ_2, can be used to approximate the models such as the φ^3, φ^4 etc. One can choose the formula (46) in such a way that Eq. are closed, see [2],

A left inverse to the operator $\hat{N}(\lambda_2)$ is given by formula:

$$\hat{N}_l^{-1} \equiv \hat{N}_l^{-1}(\lambda_2) = \int d\tilde{y}E(\tilde{y})M[\tilde{y};\hat{\eta};\lambda_2]\hat{\eta}(\tilde{y}) \tag{47}$$

with restriction $\int d\tilde{x}E(\tilde{x})H(\tilde{x}) = 1$. To see the above statements we have to use the Cuntz relations (6). It is also interesting that for a nonpolynomial operator (44) the variables $\tilde{x}$ can be continuous. For other choices of the formula (47), see [4]. See also [3].

5 Equations with different operators $\hat{N}(\lambda_2)$. The closure principle

We consider the model with the left invertible operator:

$$\hat{N}(\lambda_2) = \int d\tilde{x}\hat{\eta}^\star(\tilde{x})\left(\hat{I} + \lambda_2\hat{\eta}(\tilde{x})\right) + \lambda_2\hat{P}_0 =$$
$$\lambda_2\hat{I} + \int d\tilde{x}\hat{\eta}^\star(\tilde{x}) \equiv \lambda_2\hat{I} + \hat{G}' \tag{48}$$

In the vector form, Eq.9, for n-pi, are:

$$\left(\hat{L} + \lambda_1\hat{N}(\lambda_2) + \hat{G}\right)|V>= \hat{P}_0 + \lambda_2\hat{P}_0 = |0;\lambda_2>_{info} \tag{49}$$

or

$$\left\{\left(\hat{L} + \lambda_1\lambda_2\hat{I}\right) + \left(\hat{G} + \lambda_1\hat{G}'\right)\right\}|V>= |0;\lambda_2>_{info} \tag{50}$$

These equations can be solved term by term because the equations are not branching. Using however a left inverse operator

$$\hat{N}_l^{-1}(\lambda_2) = \int d\tilde{y}H(\tilde{y})\left(\hat{I} + \lambda_2\hat{\eta}(\tilde{y})\right)_l^{-1}\hat{\eta}(\tilde{y}) \tag{51}$$

with $\int d\tilde{x}H(\tilde{x}) = 1$, multiplying Eq.50 by this operator, we get, for n-pi, highly branching system of equations:

$$\left\{\hat{N}_l^{-1}(\lambda_2)\left(\hat{L} + \hat{G}\right) + \lambda_1\hat{I}\right\}|V>= \hat{N}_l^{-1}(\lambda_2)|0;\lambda_2>_{info}= 0 \tag{52}$$

if the operator $\hat{N}_l^{-1}(\lambda_2)$ in Eq.52 is related in some way to the formulas

$$\left(\hat{I} + \lambda_2\hat{\eta}(\tilde{y})\right)_l^{-1} \Leftrightarrow \left(\hat{I} + \lambda_2\hat{\eta}(\tilde{y})\right)^{-1} \Leftrightarrow \sum_{n=0}^{\infty}(-1)^n\left(\lambda_2\hat{\eta}(\tilde{y})\right)^n \tag{53}$$

The closure problem in the above model is artificially iniciated by an inversion of the left invertible operator $\hat{N}(\lambda_2)$.

The model with left invertible operator:

$$\hat{N}(\lambda_2) = \int d\tilde{x}\hat{\eta}^\star(\tilde{x})\left(\hat{I} + \lambda_2\hat{\eta}(\tilde{x})\right)_R^{-1} + \hat{P}_0 \tag{54}$$

leads to closed equations for n-pi, if a right inverse operator appearing in the formula (54) is defind as follws:

$$\left(\hat{I} + \lambda_2\hat{\eta}(\tilde{y})\right)_R^{-1} = \left(\hat{I} + \left(\lambda_2\hat{\eta}(\tilde{y})\right)_R^{-1}\right)^{-1}\left(\lambda_2\hat{\eta}(\tilde{y})\right)_R^{-1} \Leftrightarrow \left(\hat{I} + \lambda_2\hat{\eta}(\tilde{y})\right)^{-1} \tag{55}$$

where according to Cuntz relations (6),

$$\left(\lambda_2\hat{\eta}(\tilde{y})\right)_R^{-1} = \lambda_2^{-1}\hat{\eta}^\star(\tilde{y}) \tag{56}$$

(discrete case). In this case, the operator (54),

$$\hat{N}(\lambda_2) = \int d\tilde{x}\hat{\eta}^\star(\tilde{x})\frac{\lambda_2^{-1}\hat{\eta}^\star(\tilde{x})}{\hat{I} + \lambda_2^{-1}\hat{\eta}^\star(\tilde{x})} \tag{57}$$

and since the operators $\hat{\eta}^{\star}(\tilde{x})$ are lower triangular operators, Eq.9:

$$\left(\hat{L} + \lambda_1 \hat{N}(\lambda_2) + \hat{G}\right)|V>=|0;\lambda_2>_{info}$$

is closed. Now, even for the operator $\hat{G}=0$, the vacuum $|0;\lambda_2>_{info}$ gives no vanishing contributions to the generating vector |V>, constructed by, e.g., the formula (19). We can also get closed equations for n-pi in the case of operators:

$$\hat{N}(\lambda_2) = \int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})\frac{\lambda_2^{-1}\hat{\eta}^{\star}(\tilde{x})}{\hat{I}+\lambda_2^{-1}\hat{\eta}^{\star}(\tilde{x})}\hat{\eta}(\tilde{x})^k,\ k=1,2 \tag{58}$$

which for $\lambda_2 \ggg 1$ are related formally with the unit and φ^3 models.

Models based on the operators:

$$\hat{N}(\lambda_2) = \int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})\left(\hat{I}+\lambda_2\hat{\eta}^j(\tilde{x})\right)_R^{-1}\hat{\eta}^k(\tilde{x}) + \hat{P}_0,\ k=1,...,j+1 \tag{59}$$

admit a broader class of branching equations, if a similar formula to 55 is used. We can say that highly uncertain resolvent type of operators appearing in definitions of presented models, we define by using some kind of a *closure principle*, [5].

The resulting freedom in defining the theory can be removed, at least in the perturbation theory, by using the perturbation principle, see [5] and [2, 4, 5]. See, however, App.7.

6 A solution of infinite system of branching equations. An expansion in the inverse powers of the major coupling constant λ_1.

We consider now the model:

$$\hat{N}(\lambda_2) = \int d\tilde{y}H(\tilde{y})\left(\hat{I}+\lambda_2\hat{\eta}(\tilde{y})\right)_l^{-1}\hat{\eta}(\tilde{y}) \tag{60}$$

which is a **right invertible** operator with a right inverse:

$$\hat{N}_R^{-1}(\lambda_2) = \int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})\left(\hat{I}+\lambda_2\hat{\eta}(\tilde{x})\right) \tag{61}$$

In this case, in equations for n-pi, (9), integrations or summations appear, like in equations of statistical mechanics, see [6]. Eq.9

$$\begin{gathered}\left(\hat{L} + \lambda_1 \hat{N}(\lambda_2) + \hat{G}\right)|V>= \\ \hat{P}_0\left(\hat{L} + \lambda_1 \hat{N}(\lambda_2) + \hat{G}\right)|V>\equiv|0>_{info} \quad 62\end{gathered} \tag{62}$$

is an infinite system of branching equations if the expansion (53) is somehow justified. Multiplying Eq.62 by a right inverse, (61), we get an equivalent equation

$$\left\{\lambda_1^{-1}\hat{N}_R^{-1}(\lambda_2)\left(\hat{L}+\hat{G}\right)+\hat{I}\right\}|V>=$$
$$\lambda_1^{-1}\hat{N}_R^{-1}(\lambda_2)|0>_{info}+\hat{P}_N(\lambda_2)|V> \tag{63}$$

where the projector $\hat{P}_N(\lambda_2) = \hat{I} - \hat{N}_R^{-1}(\lambda_2)\hat{N}(\lambda_2)$ projects on the null space of the operator $\hat{N}(\lambda_2)$. The vector projection, $\hat{P}_N(\lambda_2)|V>$, is an arbitrary element of the general solution to Eq.62. It can be identified by means of the perturbation theory, with the perturbation parameter λ_2, by means of which the unknown vector |V> is expanded as:

$$|V>\equiv|V(\lambda_2)>=\sum\lambda_2^n|V>^{(n)} \tag{64}$$

It is interesting to notice that in this model, the n-th approximation $|V>^{(n)}$, after inversion of the diagonal operator $\left(\int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})\int d\tilde{y}H(\tilde{y})\hat{\eta}(\tilde{y})\right)$, is expressed by the lower order approximations at the arbitrary value of the coupling constant λ_1.

7 Final remarks; Ockham's razor, the disappearance of the demarcation line between $\hat{L}$ and $\hat{N}$ operators

The presented work shows that in classical theories with some information losses caused by smoothing data, the local information vacuum described by the vector $|0>_{info}$ exerts similar effects on the process of calculation as the quantum vacuum in the perturbation theory: Particularly, in presence of external fields, $\hat{G}\neq\hat{0}$, phenomena would go differently if the vector $|0>_{info}$would not occurred in the formulas (19), (32), or (38). Moreover, for a particular form of the operators $\hat{N}$, connected with the so called non-polynomial nonlinearities, even at $\hat{G}=0$, the classical vacuum plays an important role, if we take into account possible indeterminacy of **operator-valued functions** and define them as in Secs 4 -5, see also [5] and [2]. In fact, we can say that derived equations are one of the ways of the definition of such functions.

We recall here that appearance of this vector was the result of complement of equations on the n-pi with the zero component in such a way that the operators appearing in the equations become at least one-side invertible.

A more fundamental vector in FFS is the vector |0> by means of which (local) n-pi about an arbitrary system is created. It looks as if |0> contains information about the whole Universe. In this interpretation *intriguing is* that the vector $|0>_{info}$ describing the n-pi (quasi-local info) and the vector $|0>$, describing Total information (Global Info) about the Universe, are proportional to each other. It looks as if the FFS have contained a metaphysical inspirations!

By introducing of additional parameters, new insights into the closure problem is also obtained. In *Sec.5* it is shown how easy closed equations can be converted into infinite system of branching equations, which are difficult to solve. On the other hand, by introducing the regularization parameter, λ_3, the closed equations can be derived for the large class of branching (not closed) equations, for n-pi, Sec.3.3. The problem that then arises is the emergence of undetermined projected vector $\hat{P}_N(\lambda_2)|V>$, see Eqs (40) and (63), which belongs to a larger subspace. Derived relation (32) and the principle of perturbation suggest a choice, see also [7]. Of course, in all formulas, like in (19), the symmetry condition (17) can be used.

The lesson that seems to result from this and previous works, especially with [5], is that from only the standpoint of equations and approximations, it is more useful to operate with a more complex nonpolynomial interactions, which approximate the more simple polynomial interactions. Could be it a sign of the emergence of a new paradigm? The analogy that here arises to me is similar to replacing of one computer - with many - working together (cloud computation). Otherwise, nonlinearities, which appear at the lower level of description, need not to have polynomial forms. Nevertheless, we should still keep in mind **Ockham's rezor** law called also law of economy or law of parsimony that - Pluralitas non est ponenda sine necessitate; 'Plurality should not be posited without necessity'. The principle gives precedence to simplicity; of two competing theories, the simplest explanation of an entity is to be preferred. The principle is also expressed 'Entities are not to be multiplied beyond necessity.' In the present work, this principle is implemented in Sec.3 by using the same operators for different values of parameters λ.

There is another noteworthy fact, namely, using only formulas with one-sided reversibility. The fact that such property of operators is associated with the fundamental premise of set theory, namely the Axiom of Choice, raises hope that the derived formulas are not just formal expressions.

Another important feature of this and previous papers is blurring the differences between the operators $\hat{L}$ and $\hat{N}$. This is manifested in the fact that in the transformation equations and expansions inverse operators to both of these operators occur. Somewhat similar phenomenon has been present in the very interesting *homotopy analysis method*, see e.g. [8]. In this method the *embedding parameter* $q \in [0,1]$, or rather the *weighting factor* of relevancy of operators $\hat{L}$ and $\hat{N}$, is introduced to Eq.9 - to get

$$\left((1-q)\hat{L} + \lambda q\hat{N} + \tfrac{1}{2}\hat{G}\right)|V>=$$
$$\hat{P}_0(1-q)\hat{L}|V> + \lambda q\hat{P}_0\hat{N}|V> + \tfrac{1}{2}\hat{P}_0\hat{G} \equiv |0;q>_{info} \tag{65}$$

For $q = \frac{1}{2}$, the original Eq.9 is obtained. In this case, we can use, for example, the **expansion with respect to the weighting factor q**:

$$|V>=\sum_j q^j|V>^{(j)} \tag{66}$$

in which the j-th order terms, $|V>^{(j)}$, are expressed by the (j-1)-th. In such formulae the both operators, $\hat{L}$and $\hat{N}$, enter in a similar way. You can see what it gives, when Eq.65, with the expansion parameter q, we transform as we did with the Eq.9, with the expansion parameter λ. In the case of Eq.65, we get, for example:

$$\left(\hat{I}+\lambda\tfrac{q}{1-q}\hat{L}_R^{-1}\hat{N}+\tfrac{1}{1-q}\hat{L}_R^{-1}\hat{G}\right)|V>= \\ \tfrac{1}{1-q}\hat{L}_R^{-1}|0;q>_{info}+\hat{P}_L|V> \tag{67}$$

In this way the parameter q should enter the perturbative theory, if we want the operators $\hat{L}$ and $\hat{N}$ in the Eq.65 were included approximately with the same weight, for $q\approx 1/2$. And this is a sence of the homotopy approach.

Another similarity in treatment of the operators $\hat{L}$ and $\hat{N}$ is that |V>, for $q=\{0,1\}$, satisfies the following equations:

$$\left(\hat{L}+\frac{1}{2}\hat{G}\right)|V>=\hat{P}_0\hat{L}|V>+\frac{1}{2}\hat{P}_0\hat{G} \tag{68}$$

and

$$\left(\lambda\hat{N}+\frac{1}{2}\hat{G}\right)|V>=\lambda\hat{P}_0\hat{N}|V>+\frac{1}{2}\hat{P}_0\hat{G} \tag{69}$$

and perhaps, 'this is it'! I believe that this or similar treatment of operators $\hat{L}$ and $\hat{N}$ in the proposed approximations to the vector |V> extend their effectivness. For example, in the case of Eq.9 we get the expansion (65) in which the expansion parameter, $q=1/2<1$, corresponds to the original theory (9). Another example is the closed equations derived in Sec.3.3, and so on. We can say that due to vanishing of the demarcation line between operators $\hat{L}$ and $\hat{N}$, new possibilities are open. Also by giving the same weight of importancy of operators L and N perhaps it is in some sense an implementation of the Ockham's razor law.

Now I would like to write a few sentences about the opposite phenomenon, namely the desirability of the presence of demarcation lines in a certain areas of the theory or its equations. This is when we want to take advantage of their various properties such as algebraic, geometric, topological, etc., in order to achieve satisfactory solutions. I believe that this type of strategy is made possible by using the FFS and n-pi:)

8 App.1 about equations, components of the generating vector $|V>$ and smoothed and nonsmoothed solutions to Eq.71

We assume that these components have the following interpretation expressed by the equalities:

$$V(\tilde{x}_{(n)}) =< \varphi(\tilde{x}_1)\cdots\varphi(\tilde{x}_n) > \tag{70}$$

where the field $\varphi(\tilde{x})$ is a general solution of the 'micro'or local equation:

$$L[\tilde{x};\varphi] + \lambda N[\tilde{x};\varphi] + G(\tilde{x}) = 0 \tag{71}$$

with linear (L), nonlinear (N) and external (G) parts. The symbol $<>$ means operation of averaging or smoothing of solutions to Eq.71.

$<...>$ can also mean what does the mind during our vision and in general to survive, see App.4.

We have

$$\varphi(\tilde{x}) = \varphi[\tilde{x};\alpha] \tag{72}$$

where αdenots all the boundary and initial conditions related to Eq.71. In our case, these are *'random'* variables. So, with a probability density or a weight density, we get

$$< \varphi(\tilde{x}_1)\cdots\varphi(\tilde{x}_n) >= \int \delta\alpha\varphi[\tilde{x}_1;\alpha]\cdots\varphi[\tilde{x}_n;\alpha]W[\alpha] \tag{73}$$

were the symbol $\delta\alpha$ means the functional integration (ensemble or statistical smoothing).

There is also another possible way of smoothing the field φ by using one or a few multiple integrals. It is so called the moving average, also called the rolling average. In this case we have:

$$< \varphi(\tilde{x}_1)\cdots\varphi(\tilde{x}_n) >= \int dw\varphi[\tilde{x}_1;\alpha_w]\cdots\varphi[\tilde{x}_n;\alpha_w]W(w) \tag{74}$$

where $\alpha_w(\tilde{x}) = \alpha(\tilde{x} - \tilde{w})$, see also App.6. Naturally, if Eq.71 is not translationally invariant, we can not use the formula (74), if we hope to get the same Eq.9. In General Relativity, there is additional difficulty related to the formula (74) caused by the fact that you can not add tensors defined at different space-time points (averaging problem), see e.g., [22]. In the case of (73) we make averaging/smoothing of tensors selected at the same points.

In both cases, e.g., for

$$W(w) = \delta(w) \tag{75}$$

the formalism presented can be used, at least for certain nonlinearities, $N[\tilde{x};\varphi]$, to construct nonaveraged solutions to Eq.71. In the case of Eq.73 we can use the weight density

$$W[\alpha] = \delta[\alpha - \alpha_0] + W'[\alpha] \tag{76}$$

with the functional Dirac's δ and a smooth nonsingular functonal $W'[\alpha]$. Then, the whole set of vectors $\{|V; W', \alpha_0 >\}_{W' \subset smooth\, functionals}$ can be constructed. With the help of different α_0, the whole **class** (collection, family) of such **sets** can also be constructed. From AC a set of vectors $|V>$constituated from exact solutions to Eq.71 and their products exists. This statement can have non-trivial value if the existence of particular sets of the collection would be an easier task to prove than separate solutions.

It is also interesting to notice that smoothing/averaging operation $<...>$, at fixed constatnts of motion of Eq.9, do not change Eq.71 even so it correspons to reduction of the number of freedom.

9 App.2 about the Axiom of Choice (AC)

If we define a choice function: *a choice function on a class H of nonempty sets* is a map f with domain H such that $f(X) \in X$ for every $X \in H$, then AC can be formulated as:

'Any class of nonempty sets has a choice function.

Finally AC is easily shown to be equivalent (in the usual set theories) to:

Any surjective function has a right inverse.' (Stanford Encyclopedia of Philosophy (Internet)

xxxxx

Generalisation:

'A class is a collection of sets: for any property, we can form a class of all sets with property P. But there is no surjection from a set to a class that itself is not a set. Every set is a class. A conglomerate is collection of classes: for any property P, we can form a conglomerate of classes with property P. Moreover, we assume an AC for Conglomerates: for each surjection $f : X \to Y$ of conglomerates, there exists an injection $g : Y \to X$ with $f \odot g = idY$, the identity of Y. Every class is also a conglomerate.' ([14]).

xxxxx

Comments:

App.2 means also that right invertible operators (surjective functions) considered in the paper can transform the FFS X in another FFS Y.

From AC also results that if we have a relation defined as

$$x \sim y \quad iff \quad f(x) = f(y) \tag{77}$$

then $g \odot f$ (a projector) maps everything in each equivalence class to one of its members.

10 App.3 about complex and complicated systems. A consilience in action

'The distinction made is that a *complicated system* is composed of **many parts**, which can be seen as independent of the other parts. Sort of like in a car, if you remove the front seat, the car can still run and the general functionality of the car is left unaffected. But in a *complex system*, the components are interconnected in a way such that affecting on member will in turn affect the functionality of all members contained within the system. In a car, removing the engine belt would in turn stop the car from functioning.'

quotation from http://cuzzopaint.in/post/5012868509/complexity-complication.

The above definitions of complex and complicated systems are used in economy and may be better understood in terms of control: 'The **complex phenomena can be controlled because they are reducible to a simpler description, so that the seemingly different events can be treated similarly**'. This is quotation from [15].

If the system is described by continuous or almost continuous field, you could probably say that it consists of many parts. If these parts are somehow interconnected, as in our case by Eq.71, one can speak of a complex system. If not, and this is often related to the linear term (see Quantum Field Theory), we can speak about a complicated system. In practical economics, it is not possible to clearly separate complexity from comlication (Ludvig von Mises, Friedrich Hayek), [15]. In the theory of evolution, the situation is similar and it is conceivable that complication may reveal complexity, [15]. See also [16] for 'consilient' or 'vertically integrated' approach to the study.

It turns out that without definite knowledge concerning Eq.71 one can say something a bit about its averaged or smoothed solutions, which, in any case of non-linear theory, are not solutions to that equation. In the introduction of previous work, [4], we mentioned the possibility of using this approach in a wide range of human activity. If the field φ describes positioning of letters of a novel and operation $< ... >$ some kind of summarizing of text fragments, we can hope that higher order of n-pi can describe meaning of words or sentences. Even the reading is characterized by the property which is common feature of many mathematical operations: It strongly depends on the direction of the reading. It is not excluded that the multi-level description of the mind and body, see [18], will someday be reflected in the considered here equations for the generating vector $|V>$. After a cursory reading of the paper [17], our optimism is even greater!

Another example of consilience phenomenon is relation between memory and continuity. Greater precision 'in thinking about thinking' is also the result of 'consilience in action' in the case of computer science, linguistics, cybernetics and psychology, see [18].

At the end, let me quotate Jerome Iglowitz's paper (The Mind-Brain Problem: An Introduction for Beginners): '[Hilbert's] revolution lay in the stipulation that the basic or primitive concepts are to be defined just by the fact that they satisfy axioms...[They] acquire meaning only by virtue of the axiom system, and possess only the content that it bestows upon them. They stand for entities whose whole being is to be bearers of the relations laid down by the system'. And one more quotation from the same paper this time by the Hilbert: 'If one is looking for other definitions of A 'point', e.g. through paraphrase in terms of extensionless, etc., then I must indeed oppose such attemps in the most decisive way; one is looking for something one can never find because there is nothing there; and everything gets lost and becomes vague and tangled and and degenerates into a game of hide and seek.'

It is not excluded that the sentence: 'there is no mind' resulting from the above quotations is not purelly materialistic but it is only expression of very complex phenomena which we call the mind.

11 App.4 about a rejection and the mind

'Vision is not a simple reflection of (shooting) environment. It involves the continuous exploration and **rejection** of what the mind considers it unnecessary.' from [19].

It is believed that the selection process led to the survival of the minds, which especially effectively were avoiding superfluous information, see also [20]. In this way a person acquired the so-called opportunity to act (affordances), see J.J. Gibson and M. Błaszak in [20]. Getting rid of the huge amount of information overload would require large amounts of energy which would have a negative impact on reproduction and survival chances of organisms.

12 App.5 about a possible interpretation of explicitly appearing constants in Eq.71. A more fundamental equations?

Eq.9 can be also used in the case when constants appearing in this equation or Eq.71 are 'random' quantities. In such cases we have to solve first Eq.9 and after that we can use the averaging operation $<...>$ now with respect to these constatants. It would be interesting to investigate wether all these constants can be treated as values of corresponding integrals of motion or constants of motion of an appropriately generalized Eq.71. It would be also interesting to find whether such constants of motion are related to some symmetries like in Noether's theorem (1918), see [21]. In this way, constants, especially the fundamental constants are treated as indications that in the background of Eq.71 lie in a more simple equation. This belief stems from the fact that taking into account in Eq.71 the available in sequence - constants of motion - leads to increasing complexity of equations to a smaller number of degrees of freedom. Particularly desirable would be an equation in which the role of the eliminated degrees of freedom would amount to maintain the stability of fundamental constants. It is also possible that these additional variables could be used to describe the unexplained phenomena of the dark material world!?

By the way, I note that in the case of equations with the constants of motion due to the corresponding symmetry, it is easy to implement the inverse program outlined above. see in particular the end of App.1.

13 App.6 about smoothing or averaging the linear Eq.(71). Ergodic case.

As we know from this or other papers, the freedom of a given theory is usually reduced to its linear part. So, it is useful to discuss the case of linear Eq.71. From that point of view, we assume that equations are linear and homogeneous. In this case we denote the general solution to such equations as

$$\varphi(\tilde{x}) = \varphi_0[\tilde{x}, \alpha] \tag{78}$$

Due to the superposition principle satisfied by solutions of any linear equation, if $\varphi_0[\tilde{x}, \alpha_i]$ denote varies solutions of the linear Eq.71 satisfying varies additional conditions α_i, then their arbitrary combination:

$$\sum_i c_i \varphi_0[\tilde{x}, \alpha_i] \equiv \varphi_0[\tilde{x}, \alpha] \tag{79}$$

is also a solution of the same linear Eq.71. The same can be said of the linear combinations of functions:

$$\sum_i c_i \varphi_0[\tilde{x} + \tilde{w}_i, \alpha] \equiv \varphi_0[\tilde{x}, \alpha'] \tag{80}$$

where the additional condition (AD), α, α' in Eqs79,80 also depend on the set of coefficients c_i The linear combinations of solutions in the sum (79, 80) can be treated as prototypes of corresponding averages" discussed in App.1. Ergodicity means that these two types of averages should be equal to each other. Of course, we can only hope that ***asymptotic ergodicity*** takes place, which here means that *asymtotes of solutions are independent of initial conditions.* We mean by this that, for sufficiently large time, the values of solutions can be found in a particular set.

Of course, if nonlinearity is taken into account in Eq.71, then in averaged/smoothed solutions described by Eq.9 a dependence on the method of averaging/smoothing can be seen.

For an excellent illustration of the ergodicty idea also in the social sciences, see Wikipedia: What Is Ergodicity?".

Can we inferre from the above properties of linear equations that if solutions oscillate around some regular functions, either of these functions or other regular functions are also solutions to these linear equations? It seems that averaging or smoothing processes of solutions of a linear equation lead to more smooth solutions of the same equation if we consider linear equations with constant coefficients. In this case both kind of averaging/smoothing can be executed. The case of linear equations with variable coefficients does not allow to make averages with the help of formula (80). Such linear equations (71) (with $\hat{N} = 0$) sometimes are associated with nonlinear equations, see App.7. In such cases, it is not excluded that $< \varphi(\hat{x}) >$ is no more regular than $\varphi[\hat{x}; \alpha]$, for a certain set of additional conditions α , but agregated" quantity $< \varphi(\hat{x}) >$ better reflects human capabilities.

14 App.7 Nonlinear expressed by linear equation solutions

The Riccati equations are simple example of a situation in which the linear equations are related, by certain change of variables, to the nonlinear equations. The general case of such situation, expressed in terms of solutions, can be described by the following functional equation:

$$\varphi[\hat{x}; \alpha] = F[\hat{x}; \psi[\bullet; \alpha]] \tag{81}$$

in which the functions $\psi[\hat{y}; \alpha]$ satisfies a linear equation. We see that now all n-pi related to the fields $\varphi[\hat{x}; \alpha]$ can be constructed by means of the n-pi related to the fields $\psi[\hat{y}; \alpha]$, if the Volterra series:

$$\varphi[\hat{x}; \alpha] = F[\hat{x}; \psi[\bullet; \alpha]] = \sum_n \int d\hat{x}_{(n)} F(\hat{x}; \hat{x}_{(n)}) \psi[\hat{x}_1; \alpha] \cdots \psi[\hat{x}_n; \alpha] \tag{82}$$

is used. We can get such series if Eq.71 is solved by using a perturbation theory. We must point out here that the relation (82) depends on the way in which the perturbation parameter q is introduced, see Eq.65. We must also say that the similar relation, which connects a solution of nonlinear equation solution (NES) with linear (LES) in the case of Riccati equations:

$$NES = NES_1 + \frac{1}{LES} \tag{83}$$

where NES_1- denotes a particular solution of the nonlinear Ricci's equation, is a warning against too great deal of trust in relationships (82). The most surprising is reverse situation expressed by the equation:

$$LES = \frac{1}{NES - NES_1} \tag{84}$$

which admits the expansion (82) of LES with respect to NES! That would be yet another argument for seeking unperturbative approaches.

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