The expansion of Rindler coordinate theory with the initial velocity

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ABSTRACT

In the general relativity theory, understand Rindler coordinate theory that used the tetrad and it expand to be the new Rindler coordinate theory of the accelerated observer that has the initial velocity.

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I.Introduction

This theory is that study the Rindler coordinate theory and understand the Rindler coordinate theory and expand to be the Rindler coordinate theory of the accelerated observer that have the initial velocity.

Finding the Rindler's coordinate theory , use following the formula about the constant accelerated matter.

$$x+\frac{c^2}{a_0}=\frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c}), t=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \quad (1)$$

x and t is the coordinate and the time in the inertial system about the constant accelerated matter. a_0 is the constant acceleration, τ is invariable time about the constant accelerated matter, c is light speed in the inertial system in the free space-time.

It expand to be the Rindler coordinate theory of the accelerated observer that have the initial velocity.

The formula about 2-Dimension inertial coordinate system $S(t,x)$ and $S'(t',x')$ is

$$V=\frac{u+v_0}{1+\frac{u}{c^2}v_0} \; , \; V=\frac{dx}{dt}, u=\frac{dx'}{dt'} \; , \; dx=\frac{dx'+v_0dt'}{\sqrt{1-\frac{v_0^2}{c^2}}}, \; dt=\frac{dt'+\frac{v_0}{c^2}dx'}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}), \; a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (1\text{-}1)$$

The velocity V has the initial velocity v_0 and the velocity u is the velocity by the pure acceleration a'.

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}})=\frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{v_0^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}})=\frac{1}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{u^2}{c^2}}})$$

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (1\text{-}2)$$

In this time , if the pure acceleration a' of the velocity u is

$$a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \; , \; u=\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \quad (1\text{-}3)$$

Eq(1-2) is

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}})=a'+v_0\frac{d}{dt'}(\sqrt{1+\frac{1}{c^2}[\int a'dt']^2})$$

$$=a'+v_0\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}}\frac{a'}{c^2}=a'(1+\frac{v_0}{c^2}\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}})$$

$$=a'(1+\frac{v_0}{c^2}u) \qquad (1\text{-}4)$$

Therefore, the acceleration a about the accelerated matter that has the initial velocity v_0 in 2-Dimension inertial coordinate system $S(t,x)$ and the other acceleration a' about the accelerated matter that has not the initial velocity v_0 in 2-Dimension inertial coordinate system $S'(t',x')$ are same. In this time, if the acceleration a' is the constant acceleration a_0, the inertial acceleration in 2-Dimension inertial coordinate system $S(t,x)$ and in 2-Dimension inertial coordinate system $S'(t',x')$ is the constant acceleration a_0.

$$a_0=a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})=a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}) \quad (1\text{-}5)$$

Hence, Eq(1) is in the 2-Dimension inertial coordinate system $S'(t',x')$

$$x'=\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)$$

$$t'=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \qquad (1\text{-}6)$$

Therefore, in the 2-Dimension inertial coordinate system $S(t,x)$

$$t=\gamma(t'+\frac{v_0}{c^2}x')=\gamma(\frac{c}{a_0}\sinh(\frac{a_0}{c}\tau)+\frac{v_0}{a_0}(\cosh(\frac{a_0}{c}\tau)-1))$$

$$x=\gamma(x'+v_0t')=\gamma(\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)+\frac{v_0c}{a_0}\sinh(\frac{a_0\tau}{c})),\gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (1\text{-}7)$$

$$dt=\gamma(\cosh(\frac{a_0}{c}\tau)+\frac{v_0}{c}\sinh(\frac{a_0}{c}\tau))d\tau,$$

$$dx=\gamma(c\sinh(\frac{a_0}{c}\tau)+v_0\cosh(\frac{a_0}{c}\tau))d\tau,$$

$$V=\frac{dx}{dt}=(c\tanh(\frac{a_0}{c}\tau)+v_0)/(1+\frac{v_0}{c}\tanh(\frac{a_0}{c}\tau)),\gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad (1\text{-}8)$$

In this situation, if $a_0 \to 0$, the Rindler coordinate theory does the special relativity theory.

$$t=\gamma[c\lim_{a_0\to 0}\frac{\sinh(\frac{a_0}{c}\tau)}{a_0}+v_0\lim_{a_0\to 0}\frac{\cosh(\frac{a_0\tau}{c})-1}{a_0}]$$

$$=\gamma[c\lim_{a_0\to 0}\cosh(\frac{a_0}{c}\tau)\frac{\tau}{c}+v_0\lim_{a_0\to 0}\sinh(\frac{a_0\tau}{c})\frac{\tau}{c}]$$

$$=\frac{\tau}{\sqrt{1-\frac{v_0^2}{c^2}}}\quad,\ \gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad (1\text{-}9)$$

$$x=\gamma[c^2\lim_{a_0\to 0}\frac{\cosh(\frac{a_0\tau}{c})-1}{a_0}+v_0c\lim_{a_0\to 0}\frac{\sinh(\frac{a_0\tau}{c})}{a_0}]$$

$$=\gamma[c^2\lim_{a_0\to 0}\sinh(\frac{a_0}{c}\tau)\frac{\tau}{c}+v_0c\lim_{a_0\to 0}\cosh(\frac{a_0\tau}{c})\frac{\tau}{c}]$$

$$=\frac{v_0\tau}{\sqrt{1-\frac{v_0^2}{c^2}}}\quad,\ \gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad (1\text{-}10)$$

If $v_0 \to 0$,

$$t=\lim_{v_0\to 0}\gamma[\frac{c}{a_0}\sinh(\frac{a_0\tau}{c})+\frac{v_0}{a_0}(\cosh(\frac{a_0\tau}{c})-1)]=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}),$$

$$x=\lim_{v_0\to 0}\gamma[\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)-1)+\frac{v_0c}{a_0}\sinh(\frac{a_0\tau}{c})]=\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1),\gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

(1-11)

II. Additional chapter-I

The tetrad $e_a{}^{\mu}$ is the unit vector that is each other orthographic and it used the following formula.

$$e_a{}^{\mu}e_b{}^{\upsilon}g_{\mu\upsilon}=\eta_{ab} \quad (2)$$

$e^a{}_{\mu}$ is

$$e^a{}_{\mu}=\eta^{ab}g_{\mu\upsilon}e_b{}^{\upsilon} \quad (3)$$

and it is $e_a^{\ \mu}$'s inverse-metrix.And it is

$$e^a{}_\mu e_b^{\ \mu} = \delta^a{}_b \ , \ e^a{}_\mu e_a^{\ \upsilon} = \delta_\mu^{\ \upsilon}$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \qquad (4)$$

According to the tetrad $e^a{}_\mu$,the flat Minkowski space's 4-Dimension inertial coordinate system $S(t, x, y, z)$ transform the 4-Dimension accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ .In this time,the accelerated observer of the 4-Dimension accelerated system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ is same the accelerated matter that has the initial velocity v_0 in 2-Dimension inertial coordinate system $S(t, x)$ .Therefore

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon \quad (5)$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad (6) \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \quad (7)$$

$e^\alpha{}_\mu(\tau)$ is the tetrad that if $\xi^1 = \xi^2 = \xi^3 = 0$, $d\xi^1 = d\xi^2 = d\xi^3 = 0$ .It is not the accelerated system and it is the point's the accelerate motion. Therefore $\xi^0 = \tau$, in this case, it does $g_{\mu\upsilon} = \eta_{\mu\upsilon}$,According to Eq (1-7),Eq (6),Eq(7)

$$e^\alpha{}_0(\tau) = \frac{\partial x^\alpha}{c\partial \xi^0} = \frac{1}{c}\frac{dx^\alpha}{d\tau}$$

$$= (\gamma\cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\sinh(\frac{a_0}{c}\tau),0,0) \ , \gamma\sinh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\cosh(\frac{a_0}{c}\tau),0,0) \ , \gamma = \frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}$$

(8)

About y -axis's and z -axis's orientation

$e^\alpha{}_2(\tau) = (0,0,1,0)$ (9), $e^\alpha{}_3(\tau) = (0,0,0,1)$ (10)

And the other unit vector $e^\alpha{}_1(\tau)$ has to satisfy the tetrad condition, Eq (4)

$$e^\alpha{}_1(\tau) = (\gamma\sinh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\cosh(\frac{a_0}{c}\tau),$$

$$\gamma\cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\sinh(\frac{a_0}{c}\tau),0,0) \ , \gamma = \frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}} \qquad (11)$$

According to the accelerated system, $e^\alpha{}_\mu(\xi^0)$ is used by Eq (9),Eq (10),Eq(11) that used ξ^0 instead of τ .

The unit vector $e^{\alpha}{}_{1}(\xi^0)$ is

$$e^{\alpha}{}_{1}(\xi^0) = \frac{\partial x^{\alpha}}{\partial \xi^1} = (\gamma \sinh(\frac{a_0}{c}\xi^0) + \frac{v_0}{c}\gamma \cosh(\frac{a_0}{c}\xi^0),$$

$$\gamma \cosh(\frac{a_0}{c}\xi^0) + \frac{v_0}{c}\gamma \sinh(\frac{a_0}{c}\xi^0),0,0)\ ,\gamma = \frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \quad (12)$$

About y -axis's and z -axis's orientation, the unit vector $e^{\alpha}{}_{2}(\xi^0)$ and $e^{\alpha}{}_{3}(\xi^0)$ is

$$e^{\alpha}{}_{2}(\xi^0) = \frac{\partial x^{\alpha}}{\partial \xi^2} = (0,0,1,0) \quad (13),\quad e^{\alpha}{}_{3}(\xi^0) = \frac{\partial x^{\alpha}}{\partial \xi^3} = (0,0,0,1) \quad (14)$$

In the Rindler coordinate theory, the coordinate transformation of the inertial system x^{α} and the accelerated system ξ^{α} is

$$x^{\alpha} = \xi^1 e^{\alpha}{}_{1}(\xi^0) + \xi^2 e^{\alpha}{}_{2}(\xi^0) + \xi^3 e^{\alpha}{}_{3}(\xi^0) + z^{\alpha}(\xi^0) \quad (15)$$

ξ^1,ξ^2,ξ^3 is the accelerated system ξ^{α}'s coordinate, $z^{\alpha}(\xi^0)$ is the distance that is the origin of the accelerated system ξ^{α} in the inertial system.

If u^{α},a^{α} is

$$u^{\alpha} = \frac{dx^{\alpha}}{d\tau}, a^{\alpha} = \frac{du^{\alpha}}{d\tau} \quad (15\text{-}1)$$

The accelerated gyroscope's spin vector s^{α} is

$$\frac{ds^{\alpha}}{d\tau} = \frac{1}{c^2}(u^{\alpha}a^{\beta} - a^{\alpha}u^{\beta})s_{\beta} = \frac{1}{c^2}u^{\alpha}a^{\beta}s_{\beta} \,(15\text{-}2),\quad u^{\alpha}s_{\alpha} = 0\ ,\ s^{\alpha} = (0,\vec{s}) \ (15\text{-}3)$$

and

$$\frac{de^{\alpha}{}_{a}(\tau)}{d\tau} = \frac{1}{c^2}(u^{\alpha}a^{\beta} - a^{\alpha}u^{\beta})e_{a\beta}(\tau),\ e_{a\beta}(\tau) = \eta_{\beta\beta}e^{\beta}{}_{a}(\tau) \quad (15\text{-}4)$$

Eq(15-4) is satisfied by Eq(8),Eq(9),Eq(10),Eq(11).

Therefore, the spin vector $\vec{s}$'s orientation is the orientation of $e^{\alpha}{}_{1}, e^{\alpha}{}_{2}, e^{\alpha}{}_{3}$.

Hence, Eq(15) can use in new Rindler coordinate theory of the accelerated observer that has the initial velocity v_0.

$$z^{\alpha}(\xi^0) = (\frac{c^2}{a_0}\gamma \sinh(\frac{a_0}{c}\xi^0) + \frac{v_0 c}{a_0}\gamma(\cosh(\frac{a_0}{c}\xi^0) - 1)$$

$$,\frac{c^2}{a_0}\gamma(\cosh(\frac{a_0}{c}\xi^0) - 1) + \frac{v_0 c}{a_0}\gamma \sinh(\frac{a_0}{c}\xi^0),0,0), \gamma = \frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \quad (16)$$

Therefore if Eq (15) used by Eq (16) and Eq (12),Eq (13),Eq (14),finally the new Rindler's coordinate transformation of the accelerated observer with the initial velocity is found.

$$ct = (\xi^1 + \frac{c^2}{a_0})\gamma \sinh(\frac{a_0}{c}\xi^0) + \gamma \frac{v_0}{c}\xi^1 \cosh(\frac{a_0}{c}\xi^0)$$

$$+\gamma \frac{v_0 c}{a_0}(\cosh(\frac{a_0}{c}\xi^0) - 1)$$

$$= \gamma(\frac{c^2}{a_0} + \xi^1)\{\sinh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\} - \gamma \frac{v_0 c}{a_0} \quad (17)$$

$$x = (\xi^1 + \frac{c^2}{a_0})\gamma \cosh(\frac{a_0}{c}\xi^0) - \gamma \frac{c^2}{a_0} + \gamma \frac{v_0}{c}\xi^1 \sinh(\frac{a_0}{c}\xi^0)$$

$$+\gamma \frac{v_0 c}{a_0}\sinh(\frac{a_0}{c}\xi^0)$$

$$= \gamma(\frac{c^2}{a_0} + \xi^1)\{\cosh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\} - \gamma \frac{c^2}{a_0} \text{ (18),}$$

$$y = \xi^2, z = \xi^3 \text{ (18-1)}, \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

Therefore, the new inverse-coordinate transformation of the new Rindler theory of the accelerated observer with the initial velocity is

$$\frac{(ct + \gamma \frac{v_0 c}{a_0})}{(x + \gamma \frac{c^2}{a_0})} = \frac{\tanh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}}{1 + \frac{v_0}{c}\cdot \tanh(\frac{a_0\xi^0}{c})}$$

$$\xi^0 = \frac{c}{a_0}\tanh^{-1}[\frac{\frac{(ct + \gamma \frac{v_0 c}{a_0})}{(x + \gamma \frac{c^2}{a_0})} - \frac{v_0}{c}}{1 - \frac{v_0}{c}\cdot \frac{(ct + \gamma \frac{v_0 c}{a_0})}{(x + \gamma \frac{c^2}{a_0})}}] \text{ (18-2)}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$(x + \gamma \frac{c^2}{a_0})^2 - (ct + \gamma \frac{v_0 c}{a_0})^2 = (\frac{c^2}{a_0} + \xi^1)^2 \gamma^2 (1 - \frac{v_0^2}{c^2}) = (\frac{c^2}{a_0} + \xi^1)^2$$

$$\xi^1 = \sqrt{(x + \gamma \frac{c^2}{a_0})^2 - (ct + \gamma \frac{v_0 c}{a_0})^2} - \frac{c^2}{a_0} \quad \text{(18-3)}, \quad \xi^2 = y, \xi^3 = z \text{ (18-4)}, \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

III. Additional chapter-II

In this situation of the new Rindler's coordinate transformation of the accelerated observer with the initial velocity, if $a_0 \to 0$, the new Rindler coordinate transformation does the special relativity's coordinate transformation. Therefore, about 4-Dimension inertial coordinate system, $S(t, x, y, z)$ and $S'(t', x', y', z')$,4-Dimension accelerated system, $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$,in this time

$\xi^0 \to t', \xi^1 \to x', \xi^2 \to y', \xi^3 \to z'$

$$
\begin{aligned}
ct &= \lim_{a_0 \to 0}[\gamma(\frac{c^2}{a_0} + \xi^1)\{\sinh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\} - \gamma\frac{v_0 c}{a_0}] \\
&= \lim_{a_0 \to 0}[(\xi^1 + \frac{c^2}{a_0})\gamma\sinh(\frac{a_0}{c}\xi^0) + \gamma\frac{v_0}{c}\xi^1\cosh(\frac{a_0}{c}\xi^0) \\
&\quad + \gamma\frac{v_0 c}{a_0}(\cosh(\frac{a_0}{c}\xi^0) - 1)] \\
&= \gamma c^2 \lim_{a_0 \to 0}\frac{\sinh(\frac{a_0\xi^0}{c})}{a_0} + \frac{v_0}{c}\gamma\lim_{a_0 \to 0}\xi^1\cosh(\frac{a_0}{c}\xi^0) \\
&\quad + cv_0\gamma\lim_{a_0 \to 0}\frac{\cosh(\frac{a_0\xi^0}{c}) - 1}{a_0} \\
&= c^2\gamma\lim_{a_0 \to 0}\cosh(\frac{a_0\xi^0}{c})\frac{1}{c}\lim_{a_0 \to 0}\xi^0 + \frac{\frac{v_0}{c}\cdot x'}{\sqrt{1 - \frac{v_0^2}{c^2}}} \\
&\quad + cv_0\gamma\lim_{a_0 \to 0}\sinh(\frac{a_0\xi^0}{c})\frac{1}{c}\lim_{a_0 \to 0}\xi^0 \\
&= \frac{ct' + \frac{v_0}{c}\cdot x'}{\sqrt{1 - \frac{v_0^2}{c^2}}}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
x &= \lim_{a_0 \to 0}[\gamma(\frac{c^2}{a_0} + \xi^1)\{\cosh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\} - \gamma\frac{c^2}{a_0}] \\
&= \lim_{a_0 \to 0}[(\xi^1 + \frac{c^2}{a_0})\gamma\cosh(\frac{a_0}{c}\xi^0) - \gamma\frac{c^2}{a_0} + \gamma\frac{v_0}{c}\xi^1\sinh(\frac{a_0}{c}\xi^0) \\
&\quad + \gamma\frac{v_0 c}{a_0}\sinh(\frac{a_0}{c}\xi^0)]
\end{aligned}
$$

$$= \gamma \lim_{a_0 \to 0} \xi^1 \cosh(\frac{a_0}{c}\xi^0) + c^2 \gamma \lim_{a_0 \to 0} \frac{\cosh(\frac{a_0 \xi^0}{c}) - 1}{a_0}$$

$$+ \lim_{a_0 \to 0} \gamma \frac{v_0}{c} \xi^1 \sinh(\frac{a_0}{c}\xi^0) + v_0 c \gamma \lim_{a_0 \to 0} \frac{\sinh(\frac{a_0}{c}\xi^0)}{a_0}$$

$$= \frac{x'}{\sqrt{1 - \frac{{v_0}^2}{c^2}}} + c^2 \lim_{a_0 \to 0} \sinh(\frac{a_0 \xi^0}{c}) \frac{1}{c} \gamma \lim_{a_0 \to 0} \xi^0 + v_0 c \lim_{a_0 \to 0} \cosh(\frac{a_0 \xi^0}{c}) \frac{1}{c} \gamma \lim_{a_0 \to 0} \xi^0$$

$$= \frac{x' + v_0 t'}{\sqrt{1 - \frac{{v_0}^2}{c^2}}}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{{v_0}^2}{c^2}}} \quad (20) \quad y = \lim_{a_0 \to 0} \xi^2 = y', z = \lim_{a_0 \to 0} \xi^3 = z' \quad (20\text{-}1)$$

In this situation, if $v_0 \to 0$, the new coordinate transformation does the Rindler coordinate transformation.

$$ct = \lim_{v_0 \to 0} [\gamma (\frac{c^2}{a_0} + \xi^1) \{\sinh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \cosh(\frac{a_0 \xi^0}{c})\} - \gamma \frac{v_0 c}{a_0}]$$

$$= \lim_{v_0 \to 0} \gamma \{(\xi^1 + \frac{c^2}{a_0}) \sinh(\frac{a_0}{c}\xi^0) + \frac{v_0}{c} \xi^1 \cosh(\frac{a_0}{c}\xi^0) \qquad + \frac{v_0 c}{a_0} (\cosh(\frac{a_0}{c}\xi^0) - 1)\}$$

$$= (\xi^1 + \frac{c^2}{a_0}) \sinh(\frac{a_0}{c}\xi^0) \quad (21)$$

$$x = \lim_{v_0 \to 0} [\gamma (\frac{c^2}{a_0} + \xi^1) \{\cosh(\frac{a_0 \xi^0}{c}) + \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c})\} - \gamma \frac{c^2}{a_0}]$$

$$= \lim_{v_0 \to 0} \gamma \{(\xi^1 + \frac{c^2}{a_0}) \cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0} + \frac{v_0}{c} \xi^1 \sinh(\frac{a_0}{c}\xi^0) \qquad + \frac{v_0 c}{a_0} \sinh(\frac{a_0}{c}\xi^0)\}$$

$$= (\xi^1 + \frac{c^2}{a_0}) \cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0} \, (22), \quad \gamma = \frac{1}{\sqrt{1 - \frac{{v_0}^2}{c^2}}}, y = \xi^2, z = \xi^3 \, (22\text{-}1)$$

In Eq(17),in Eq(18),in Eq(18-1), the differential coordinate transformation of the new Rindler's coordinate theory of the accelerated observer with the initial velocity is

$$cdt = \gamma [d\xi^1 \sinh(\frac{a_0 \xi^0}{c}) + (\xi^1 + \frac{c^2}{a_0}) \cosh(\frac{a_0 \xi^0}{c}) \frac{a_0}{c} d\xi^0$$

$$+ d\xi^1 \frac{v_0}{c} \cosh(\frac{a_0}{c}\xi^0) + \xi^1 \frac{v_0}{c} \sinh(\frac{a_0 \xi^0}{c}) \frac{a_0}{c} d\xi^0$$

$$+\frac{v_0 c}{a_0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0]$$

$$=\gamma[(1+\frac{a_0}{c^2}\xi^1)\{\cosh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\}cd\xi^0$$

$$+\{\sinh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\}d\xi^1] \quad (23)$$

$$dx=\gamma[d\xi^1\cosh(\frac{a_0\xi^0}{c})+(\xi^1+\frac{c^2}{a_0})\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0$$

$$+d\xi^1\frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})+\xi^1\frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0$$

$$+\frac{v_0 c}{a_0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0]$$

$$=\gamma[(1+\frac{a_0}{c^2}\xi^1)\{\sinh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\}cd\xi^0$$

$$+\{\cosh(\frac{a_0\xi^0}{c})+\frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\}d\xi^1] \ (24), \gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}, dy=d\xi^2, dz=d\xi^3 \ (25)$$

Therefore, the invariable time $d\tau$ of the new Rindler's coordinate theory of the accelerated observer with the initial velocity is

$$d\tau^2=dt^2-\frac{1}{c^2}[dx^2+dy^2+dz^2]$$

$$=(1+\frac{a_0\xi^1}{c^2})^2(d\xi^0)^2-\frac{1}{c^2}[(d\xi^1)^2+(d\xi^2)^2+(d\xi^3)^2] \ (26)$$

Hence, the invariable time $d\tau$ of the new Rindler coordinate theory of the accelerated observer that has the initial velocity v_0 is not related to the initial velocity v_0.

You can save new Rindler's coordinate transformation of the accelerated observer that has the initial velocity v_0 in the other way.

In $S'(t',x',y',z')$ of 4-Dimension inertial coordinate system and in $\xi(\xi^0,\xi^1,\xi^2,\xi^3)$ of 4-Dimension accelerated system, the Rindler's coordinate transformation is

$$ct'=(\xi^1+\frac{c^2}{a_0})\sinh(\frac{a_0}{c}\xi^0), x'=(\xi^1+\frac{c^2}{a_0})\cosh(\frac{a_0}{c}\xi^0)-\frac{c^2}{a_0} \ (27) \quad y'=\xi^2, z'=\xi^3 \ (27\text{-}1)$$

Therefore, in $S(t,x,y,z)$ of 4-Dimension inertial coordinate system and in $\xi(\xi^0,\xi^1,\xi^2,\xi^3)$ of 4-Dimension accelerated system, new Rindler's coordinate transformation of the accelerated observer that has the initial velocity v_0 is

$$ct = \gamma(ct' + \frac{v_0}{c} \cdot x')$$

$$= \gamma\ [(\xi^1 + \frac{c^2}{a_0})\sinh(\frac{a_0}{c}\xi^0)\ + \frac{v_0}{c} \cdot \{(\xi^1 + \frac{c^2}{a_0})\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}]$$

$$= \gamma(\frac{c^2}{a_0} + \xi^1)\{\sinh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\} - \gamma\frac{v_0 c}{a_0}\ (28)\ , \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$x = \gamma(x' + \frac{v_0}{c} \cdot ct')$$

$$= \gamma\ [\{(\xi^1 + \frac{c^2}{a_0})\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}\ + \frac{v_0}{c} \cdot (\xi^1 + \frac{c^2}{a_0})\sinh(\frac{a_0}{c}\xi^0)]$$

$$= \gamma(\frac{c^2}{a_0} + \xi^1)\{\cosh(\frac{a_0\xi^0}{c}) + \frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\} - \gamma\frac{c^2}{a_0}\ (29),\ \ \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$y = y' = \xi^2, z = z' = \xi^3\ (29\text{-}1)$$

IV.Conclusion

The Rindler coordinate theory expanded to be new Rindler coordinate theory of the accelerated observer that has the initial velocity.

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