# The expansion of Rindler coordinate theory with the initial velocity

**Sangwha-Yi**

**Department of Math , Taejon University 300-716**

**ABSTRACT**

In the general relativity theory, understand Rindler coordinate theory that used the tetrad and it expand to be the new Rindler coordinate theory that has the initial velocity.

**PACS Number**:**04,04.90.+e**

**Key words:The general relativity theory,**

**The Rindler coordinate theory,**

**The initial velocity**

**The coordinate transformation,**

**The tetrad**

**e-mail address:**sangwha1@**nate.com**

**Tel:051-624-3953**

## I.Introduction

This theory is that study the Rindler coordinate theory and understand the Rindler coordinate theory and expand to be the Rindler coordinate theory that have the initial velocity.

Finding the Rindler's coordinate theory , use following the formula about the constant accelerated matter.

$$x+\frac{c^2}{a_0}=\frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c}), t=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \quad (1)$$

$x$ and $t$ is the coordinate and the time in the inertial system about the constant accelerated matter. $a_0$ is the constant acceleration, $\tau$ is invariable time about the constant accelerated matter, $c$ is light speed in the inertial system in the free space-time.

It expand to be the Rindler coordinate theory that have the initial velocity.

The formula about $S(t,x)$ and $S'(t',x')$ is

$$V=\frac{u+v_0}{1+\frac{u}{c^2}v_0} \; , \; V=\frac{dx}{dt}, u=\frac{dx'}{dt'} \; , \; dx=\frac{dx'+v_0dt'}{\sqrt{1-\frac{v_0^2}{c^2}}}, \; dt=\frac{dt'+\frac{v_0}{c^2}dx'}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}), \; a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (1\text{-}1)$$

The velocity $V$ has the initial velocity $v_0$ and the velocity $u$ is the velocity by the pure acceleration $a'$.

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}})=\frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{v_0^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}})=\frac{1}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{u^2}{c^2}}})$$

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (1\text{-}2)$$

In this time , if the pure acceleration $a'$ of the velocity $u$ is

$$a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \; , \; u=\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \quad (1\text{-}3)$$

Eq(1-2) is

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}})=a'+v_0\frac{d}{dt'}(\sqrt{1+\frac{1}{c^2}[\int a'dt']^2})$$

$$=a'+v_0\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}}\frac{a'}{c^2}=a'(1+\frac{v_0}{c^2}\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}})$$

$$=a'(1+\frac{v_0}{c^2}u) \qquad (1\text{-}4)$$

Therefore, the acceleration $a$ about the accelerated matter that has the initial velocity $v_0$ in $S(t,x)$ and the other acceleration $a'$ about the accelerated matter that has not the initial velocity $v_0$ in $S'(t',x')$ is same.

In this time, if the acceleration $a'$ is the constant acceleration $a_0$, the inertial acceleration in $S(t,x)$ and in $S'(t',x')$ is the constant acceleration $a_0$.

$$a_0=a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})=a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}) \qquad (1\text{-}5)$$

Hence, Eq(1) is in the $S'(t',x')$

$$x'=\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)$$

$$t'=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \qquad (1\text{-}6)$$

Therefore,

$$t=\gamma(t'+\frac{v_0}{c^2}x')=\gamma(\frac{c}{a_0}\sinh(\frac{a_0}{c}\tau)+\frac{v_0}{a_0}(\cosh(\frac{a_0}{c}\tau)-1))$$

$$x=\gamma(x'+v_0t')=\gamma(\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)+\frac{v_0c}{a_0}\sinh(\frac{a_0\tau}{c})),\gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (1\text{-}7)$$

$$dt=\gamma(\cosh(\frac{a_0}{c}\tau)+\frac{v_0}{c}\sinh(\frac{a_0}{c}\tau))d\tau,$$

$$dx=\gamma(c\sinh(\frac{a_0}{c}\tau)+v_0\cosh(\frac{a_0}{c}\tau))d\tau,$$

$$V=\frac{dx}{dt}=(c\tanh(\frac{a_0}{c}\tau)+v_0)/(1+\frac{v_0}{c}\tanh(\frac{a_0}{c}\tau)),\gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad (1\text{-}8)$$

In this situation, if $a_0 \to 0$, the Rindler coordinate theory does the special relativity theory.

$$t=\gamma[c\lim_{a_0\to 0}\frac{\sinh(\frac{a_0}{c}\tau)}{a_0}+v_0\lim_{a_0\to 0}\frac{\cosh(\frac{a_0\tau}{c})-1}{a_0}]$$

$$=\gamma[c\lim_{a_0\to 0}\cosh(\frac{a_0}{c}\tau)\frac{\tau}{c}+v_0\lim_{a_0\to 0}\sinh(\frac{a_0\tau}{c})\frac{\tau}{c}]$$

$$=\frac{\tau}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad , \gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (1\text{-}9)$$

$$x=\gamma[c^2\lim_{a_0\to 0}\frac{\cosh(\frac{a_0\tau}{c})-1}{a_0}+v_0c\lim_{a_0\to 0}\frac{\sinh(\frac{a_0\tau}{c})}{a_0}]$$

$$=\gamma[c^2\lim_{a_0\to 0}\sinh(\frac{a_0}{c}\tau)\frac{\tau}{c}+v_0c\lim_{a_0\to 0}\cosh(\frac{a_0\tau}{c})\frac{\tau}{c}]$$

$$=\frac{v_0\tau}{\sqrt{1-\frac{v_0^2}{c^2}}} \quad , \gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (1\text{-}10)$$

If $v_0 \to 0$,

$$t=\lim_{v_0\to 0}\gamma[\frac{c}{a_0}\sinh(\frac{a_0\tau}{c})+\frac{v_0}{a_0}(\cosh(\frac{a_0\tau}{c})-1)]=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}),$$

$$x=\lim_{v_0\to 0}\gamma[\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)-1)+\frac{v_0c}{a_0}\sinh(\frac{a_0\tau}{c})]=\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1),\gamma=\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

(1-11)

## II. Additional chapter-I

The tetrad $e_a^{\ \mu}$ is the unit vector that is each other orthographic and it used the following formula.

$$e_a^{\ \mu}e_b^{\ \upsilon}g_{\mu\upsilon}=\eta_{ab} \quad (2)$$

$e^a_{\ \mu}$ is

$$e^a_{\ \mu}=\eta^{ab}g_{\mu\upsilon}e_b^{\ \upsilon} \ (3)$$

and it is $e_a^{\ \mu}$'s inverse-metrix.And it is

$$e^a{}_\mu e_b^{\ \mu} = \delta^a{}_b \ , \ e^a{}_\mu e_a^{\ \upsilon} = \delta_\mu^{\ \upsilon}$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \qquad (4)$$

According to the tetrad $e^a{}_\mu$ ,the flat Minkowski space's inertial coordinate system transform the accelerated system $\xi$

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon \quad (5)$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad (6) \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \quad (7)$$

$e^\alpha{}_\mu(\tau)$ is the tetrad that if $\xi^1 = \xi^2 = \xi^3 = 0$ , $d\xi^1 = d\xi^2 = d\xi^3 = 0$ .It is not the accelerated system and it is the point's the accelerate motion. Therefore $\xi^0 = \tau$ , in this case, it does $g_{\mu\upsilon} = \eta_{\mu\upsilon}$ ,According to Eq (1-7),Eq (6),Eq(7)

$$e^\alpha{}_0(\tau) = \frac{\partial x^\alpha}{c\partial \xi^0} = \frac{1}{c}\frac{dx^\alpha}{d\tau}$$

$$= (\gamma\cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\sinh(\frac{a_0}{c}\tau),0,0) \ ,\gamma\sinh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\cosh(\frac{a_0}{c}\tau),0,0) \ ,\gamma = \frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}$$

(8)

About $y$ -axis's and $z$ -axis's orientation

$e^\alpha{}_2(\tau) = (0,0,1,0)$ (9), $e^\alpha{}_3(\tau) = (0,0,0,1)$ (10)

and the other vector $e^\alpha{}_1(\tau)$ has to satisfy the tetrad condition, Eq (4)

$$e^\alpha{}_1(\tau) = (\gamma\sinh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\cosh(\frac{a_0}{c}\tau),$$

$$\gamma\cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\gamma\sinh(\frac{a_0}{c}\tau),0,0) \ ,\gamma = \frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}} \qquad (11)$$

According to the accelerated system, $e^\alpha{}_\mu(\xi^0)$ is used by Eq (9),Eq (10),Eq(11) that used $\xi^0$ instead of $\tau$ .

The vector $e^\alpha{}_1(\xi^0)$ is

$$e^{\alpha}{}_{1}(\xi^{0}) = \frac{\partial x^{\alpha}}{\partial \xi^{1}} = (\gamma \sinh(\frac{a_0}{c}\xi^{0}) + \frac{v_0}{c}\gamma \cosh(\frac{a_0}{c}\xi^{0}),$$

$$\gamma \cosh(\frac{a_0}{c}\xi^{0}) + \frac{v_0}{c}\gamma \sinh(\frac{a_0}{c}\xi^{0}),0,0) , \gamma = \frac{1}{\sqrt{1-\frac{v_0^{2}}{c^{2}}}} \quad (12)$$

About $y$ -axis's and $z$ -axis's orientation

$$e^{\alpha}{}_{2}(\xi^{0}) = \frac{\partial x^{\alpha}}{\partial \xi^{2}} = (0,0,1,0) \quad (13), \quad e^{\alpha}{}_{3}(\xi^{0}) = \frac{\partial x^{\alpha}}{\partial \xi^{3}} = (0,0,0,1) \quad (14)$$

In the Rindler coordinate theory, the inertial system $x^{\alpha}$'s and the accelerated system $\xi^{\alpha}$'s the coordinate transformation

$$x^{\alpha} = \xi^{1}e^{\alpha}{}_{1}(\xi^{0}) + \xi^{2}e^{\alpha}{}_{2}(\xi^{0}) + \xi^{3}e^{\alpha}{}_{3}(\xi^{0}) + z^{\alpha}(\xi^{0}) \quad (15)$$

$\xi^{1},\xi^{2},\xi^{3}$ is the accelerated system $\xi^{\alpha}$'s coordinate, $z^{\alpha}(\xi^{0})$ is the distance that is the origin of the accelerated system $\xi^{\alpha}$ in the inertial system.

Eq(15) can use in new Rindler coordinate theory that has the initial velocity $v_0$.

$$z^{\alpha}(\xi^{0}) = (\frac{c^{2}}{a_0}\gamma \sinh(\frac{a_0}{c}\xi^{0}) + \frac{v_0 c}{a_0}\gamma(\cosh(\frac{a_0}{c}\xi^{0}) - 1)$$

$$, \frac{c^{2}}{a_0}\gamma(\cosh(\frac{a_0}{c}\xi^{0}) - 1) + \frac{v_0 c}{a_0}\gamma \sinh(\frac{a_0}{c}\xi^{0}),0,0) , \gamma = \frac{1}{\sqrt{1-\frac{v_0^{2}}{c^{2}}}} \quad (16)$$

Therefore Eq (15) used by Eq (16) and Eq (12),Eq (13),Eq (14),finally the new Rindler's coordinate transformation with the initial velocity is found.

$$ct = (\xi^{1} + \frac{c^{2}}{a_0})\gamma \sinh(\frac{a_0}{c}\xi^{0}) + \gamma\frac{v_0}{c}\xi^{1}\cosh(\frac{a_0}{c}\xi^{0})$$

$$+ \gamma\frac{v_0 c}{a_0}(\cosh(\frac{a_0}{c}\xi^{0}) - 1) \quad (17)$$

$$x = (\xi^{1} + \frac{c^{2}}{a_0})\gamma \cosh(\frac{a_0}{c}\xi^{0}) - \gamma\frac{c^{2}}{a_0} + \gamma\frac{v_0}{c}\xi^{1}\sinh(\frac{a_0}{c}\xi^{0})$$

$$+ \gamma\frac{v_0 c}{a_0}\sinh(\frac{a_0}{c}\xi^{0}) \ (18) \quad , \quad y = \xi^{2}, z = \xi^{3} \ (18\text{-}1), \quad \gamma = \frac{1}{\sqrt{1-\frac{v_0^{2}}{c^{2}}}}$$

## III. Additional chapter-II

In this situation if $a_0 \to 0$, the new Rindler coordinate transformation does the special relativity's coordinate transformation. Therefore, in this time $\xi^{0} \to t', \xi^{1} \to x', \xi^{2} \to y', \xi^{3} \to z'$

$$ct = \gamma c^2 \lim_{a_0 \to 0} \frac{\sinh(\frac{a_0 \xi^0}{c})}{a_0} + \frac{v_0}{c} \gamma \lim_{a_0 \to 0} \xi^1 \cosh(\frac{a_0}{c} \xi^0)$$

$$+ c v_0 \gamma \lim_{a_0 \to 0} \frac{\cosh(\frac{a_0 \xi^0}{c}) - 1}{a_0}$$

$$= c^2 \gamma \lim_{a_0 \to 0} \cosh(\frac{a_0 \xi^0}{c}) \frac{1}{c} \lim_{a_0 \to 0} \xi^0 + \frac{\frac{v_0}{c} \cdot x'}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$+ c v_0 \gamma \lim_{a_0 \to 0} \sinh(\frac{a_0 \xi^0}{c}) \frac{1}{c} \lim_{a_0 \to 0} \xi^0$$

$$= \frac{ct' + \frac{v_0}{c} \cdot x'}{\sqrt{1 - \frac{v_0^2}{c^2}}}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \quad (19)$$

$$x = \gamma \lim_{a_0 \to 0} \xi^1 \cosh(\frac{a_0}{c} \xi^0) + c^2 \gamma \lim_{a_0 \to 0} \frac{\cosh(\frac{a_0 \xi^0}{c}) - 1}{a_0}$$

$$+ \lim_{a_0 \to 0} \gamma \frac{v_0}{c} \xi^1 \sinh(\frac{a_0}{c} \xi^0) + v_0 c \gamma \lim_{a_0 \to 0} \frac{\sinh(\frac{a_0}{c} \xi^0)}{a_0}$$

$$= \frac{x'}{\sqrt{1 - \frac{v_0^2}{c^2}}} + c^2 \lim_{a_0 \to 0} \sinh(\frac{a_0 \xi^0}{c}) \frac{1}{c} \gamma \lim_{a_0 \to 0} \xi^0 + v_0 c \lim_{a_0 \to 0} \cosh(\frac{a_0 \xi^0}{c}) \frac{1}{c} \gamma \lim_{a_0 \to 0} \xi^0$$

$$= \frac{x' + v_0 t'}{\sqrt{1 - \frac{v_0^2}{c^2}}}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \quad (20) \quad y = \lim_{a_0 \to 0} \xi^2 = y', z = \lim_{a_0 \to 0} \xi^3 = z' \quad (20\text{-}1)$$

If $v_0 \to 0$, the new coordinate transformation does the Rindler coordinate transformation.

$$ct = \lim_{v_0 \to 0} \gamma \{ (\xi^1 + \frac{c^2}{a_0}) \sinh(\frac{a_0}{c} \xi^0) + \frac{v_0}{c} \xi^1 \cosh(\frac{a_0}{c} \xi^0) \qquad + \frac{v_0 c}{a_0} (\cosh(\frac{a_0}{c} \xi^0) - 1) \}$$

$$= (\xi^1 + \frac{c^2}{a_0}) \sinh(\frac{a_0}{c} \xi^0) \ (21)$$

$$x = \lim_{v_0 \to 0} \gamma \{ (\xi^1 + \frac{c^2}{a_0}) \cosh(\frac{a_0}{c} \xi^0) - \frac{c^2}{a_0} + \frac{v_0}{c} \xi^1 \sinh(\frac{a_0}{c} \xi^0) \qquad + \frac{v_0 c}{a_0} \sinh(\frac{a_0}{c} \xi^0) \}$$

$$= (\xi^1 + \frac{c^2}{a_0})\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0} \quad (22), \quad \gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}}, y = \xi^2, z = \xi^3 \quad (22\text{-}1)$$

In Eq(17),In Eq(18),In Eq(18-1), the differential coordinate transformation is

$$cdt = \gamma[d\xi^1 \sinh(\frac{a_0\xi^0}{c}) + (\xi^1 + \frac{c^2}{a_0})\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0$$

$$+ d\xi^1 \frac{v_0}{c}\cosh(\frac{a_0}{c}\xi^0) + \xi^1 \frac{v_0}{c}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0$$

$$+ \frac{v_0 c}{a_0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0] \quad (23)$$

$$dx = \gamma[d\xi^1 \cosh(\frac{a_0\xi^0}{c}) + (\xi^1 + \frac{c^2}{a_0})\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0$$

$$+ d\xi^1 \frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c}) + \xi^1 \frac{v_0}{c}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0$$

$$+ \frac{v_0 c}{a_0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}d\xi^0] \quad (24), \quad \gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}}, \quad dy = d\xi^2, dz = d\xi^3 \quad (25)$$

Therefore, the invariable time $d\tau$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= (1 + \frac{a_0\xi^1}{c^2})^2 (d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2] \quad (26)$$

Hence, the invariable time $d\tau$ of the new Rindler coordinate theory that has the initial velocity $v_0$ is not related to the initial velocity $v_0$.

## IV.Conclusion

The Rindler coordinate theory expanded to be new Rindler coordinate theory that has the initial velocity.

# Reference


[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]W.Rindler, Am.J.Phys.**34**.1174(1966)

[3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)