The local Doppler effect due to Earth's rotation and the CNGS neutrino anomaly due to neutrino's index of refraction through the Earth Crust

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In this brief paper, we show the neutrino velocity discrepancy obtained in [1] may be due to the local Doppler effect between a local clock attached to a given detector at Gran Sasso, say C_G, and the respective instantaneous clock crossing C_G, say C_C, being this latter at rest in the instantaneous inertial frame having got the velocity of rotation of CERN about Earth's axis in relation to the fixed stars. With this effect, the index of refraction of the Earth crust may accomplish a refractive effect by which the neutrino velocity through the Earth crust turns out to be small in relation to the speed of light in the empty space.

1 Definitions and Solution

Firstly, the effect investigated here is not the same one that was investigated in [2], but, throughout this paper, we will use some useful configurations defined in [2]. The relative velocity between Gran Sasso and CERN due to the Earth daily rotation may be written:

$$\vec{v}_G - \vec{v}_C = 2\omega R \sin\alpha \hat{e}_z, \quad (1)$$

where $\hat{e}_z$ is a convenient unitary vector, the same used in [2], ω is the norm of the Earth angular velocity vector about its daily rotation axis, being R given by:

$$R_E = \frac{R}{\cos\lambda}, \quad (2)$$

where R_E is the radius of the Earth, its averaged value $R_E = 6.37 \times 10^6 m$, and α given by:

$$\alpha = \frac{1}{2}\left(\alpha_G - \alpha_C\right), \quad (3)$$

where α_C and α_G are, respectively, CERN's and Gran Sasso's longitudes ($\leftarrow WE \rightarrow$). Consider the inertial (in relation to the fixed stars) reference frame $O_C x_C y_C z_C \equiv Oxyz$ in [2]. This is the lab reference frame and consider this frame with its local clocks at each spatial position as being ideally synchronized, viz., under an ideal situation of synchronicity between the clocks of $O_C x_C y_C z_C \equiv Oxyz$. This situation is the expected ideal situation for the OPERA collaboration regarding synchronicity in the instantaneous lab (CERN) frame.

Now, consider an interaction between a single neutrino and a local detector at Gran Sasso. This event occurs at a given spacetime point $(t_\nu, x_\nu, y_\nu, z_\nu)$ in $O_C x_C y_C z_C \equiv Oxyz$. The interaction instant t_ν is measured by a local clock C_C at rest at (x_ν, y_ν, z_ν) in the lab frame, viz., in the $O_C x_C y_C z_C \equiv Oxyz$ frame. But, under gedanken, at this instant t_ν, according to $O_C x_C y_C z_C \equiv Oxyz$, there is a clock C_G attached to the detector at Gran Sasso that crosses the point $(t_\nu, x_\nu, y_\nu, z_\nu)$ with velocity given by Eq. (1). Since C_G crosses C_C, the Doppler effect between the proper tic-tac rates measured at each location of C_C and C_G, viz., measured at their respective locations in their respective reference frames (the reference frame of C_G is the $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ in [2], also inertial in relation to the fixed stars), regarding a gedanken control tic-tac rate continuosly sent by C_C, say via electromagnetic pulses from C_C, is not transverse. Since the points at which C_C and C_G are at rest in their respective reference frames will instantaneously coincide, better saying, will instantaneously intersect, at t_ν accordingly to C_C, they must be previously approximating, shortening their mutual distance during the interval $t_\nu - \delta t_\nu \rightarrow t_\nu$ ($\delta t_\nu \rightarrow 0$, $\delta t_\nu > 0$) along the line passing through these clocks as described in the C_C world.

Suppose C_C sends $\mathcal{N}$ electromagnetic pulses to C_G. During the C_C proper time interval $(t_\nu - \delta t_\nu) - 0 = t_\nu - \delta t_\nu$ * within which C_C emits the N electromagnetic pulses, the first emitted pulse travels the distance $c\,(t_\nu - \delta t_\nu)$ and reaches the clock C_G, as described by C_C. Within this distance, there are $\mathcal{N}$ equally spaced distances between consecutive pulses as described in the C_C world, say λ_C:

$$\mathcal{N}\lambda_C = c\,(t_\nu - \delta t_\nu)\,. \quad (4)$$

Also, since the clocks C_C and C_G will intersect at t_ν, as described in $O_C x_C y_C z_C \equiv Oxyz$, during the interval δt_ν, the clock C_G must travel the distance $2\omega R \sin\alpha\,\delta t_\nu$ in the C_C world to accomplish the matching spatial intersection at the instant t_ν, hence the clock C_G travels the $2\omega R \sin\alpha\,\delta t_\nu$ in the

*The initial instant C_C starts to emit the electromagnetic pulses is set to zero in both the frames $O_C x_C y_C z_C \equiv Oxyz$ and $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$; zero also is the instant the neutrino starts the travel to Gran Sasso in $O_C x_C y_C z_C \equiv Oxyz$; hence the instant the neutrino starts the travel to Gran Sasso and the emission of the first pulse by C_C are simultaneous events in $O_C x_C y_C z_C \equiv Oxyz$. These events are simultaneous in $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ too, since they have got the same spatial coordinate $z_c = z = 0$ along the $O_C z_C \equiv Oz$ direction as defined in [2]. The realtive motion between CERN and Gran Sasso is parallel to this direction. The only one difference between these events is the difference in their $x_C = x$ coordinates, being $x_C = 0$ for the neutrino departure and $x_C = L = 7.3 \times 10^5\,m$ for C_C, being these locations perpendicularly located in relation to the relative velocity given by the Eq, (1).

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C_C world, viz., as described by C_C in $O_C x_C y_C z_C \equiv Oxyz$:

$$\mathcal{N}\lambda_C = 2\omega R \sin\alpha \,\delta t_\nu \Rightarrow \delta t_\nu = \mathcal{N}\frac{\lambda_C}{2\omega R \sin\alpha}. \quad (5)$$

Solving for t_ν, from the Eqs. (4) and (5), one reaches:

$$t_\nu = \frac{\mathcal{N}\lambda_C}{c}\left(1 + \frac{c}{2\omega R \sin\alpha}\right). \quad (6)$$

Now, from the perspective of C_G, in $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$, there must be $\mathcal{N}$ electromagnetic pulses covering the distance:

$$c\left(t_\nu^G - \delta t_\nu^G\right) - 2\omega R \sin\alpha \left(t_\nu^G - \delta t_\nu^G\right), \quad (7)$$

where $t_\nu^G - \delta t_\nu^G$ is the time interval between the non-proper instants $t^G = t_\nu = 0$, at which the C_C clock send the first pulse, and the instant $t_\nu^G - \delta t_\nu^G$, at which this first pulse reaches C_G, as described by C_G in its world $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$. Within this time interval, $t_\nu^G - \delta t_\nu^G$, C_G describes, in its $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ world, the clock C_C approximating the distance:

$$2\omega R \sin\alpha \left(t_\nu^G - \delta t_\nu^G\right), \quad (8)$$

with the first pulse traveling:

$$c\left(t_\nu^G - \delta t_\nu^G\right), \quad (9)$$

giving the distance within which there must be $\mathcal{N}$ equally spaced pulses, say, spaced by λ_G, as described by C_G in its $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ world:

$$\mathcal{N}\lambda_G = (c - 2\omega R \sin\alpha)\left(t_\nu^G - \delta t_\nu^G\right). \quad (10)$$

With similar reasoning that led to the Eq. (5), now in the $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ C_G world, prior to the spatial matching intersection between C_C and C_G, the C_C clock must travel the distance $\mathcal{N}\lambda_G$ during the time interval δt_ν^G, with the C_C approximation velocity $2\omega R \sin\alpha$:

$$\mathcal{N}\lambda_G = 2\omega R \sin\alpha \,\delta t_\nu^G \Rightarrow \delta t_\nu^G = \mathcal{N}\frac{\lambda_G}{2\omega R \sin\alpha}. \quad (11)$$

From Eqs. (10) and (11), we solve for t_ν^G:

$$t_\nu^G = \mathcal{N}\frac{\lambda_G}{2\omega R \sin\alpha}\frac{1}{[1 - (2\omega R \sin\alpha)/c]}. \quad (12)$$

From the Eqs. (6) and (12), we have got the relation between the neutrino arrival instant t_ν as measured by the CERN reference frame, $O_C x_C y_C z_C \equiv Oxyz$, and the neutrino arrival instant t_ν^G as measured by the Gran Sasso reference frame, $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$, at the exact location of the interaction at an interation location within the Gran Sasso block of detectors, provided the effect of the Earth daily rotation under the assumptions we are taking in relation to the intantaneous movements of these locations in relation to the fixed stars as previously discussed:

$$\frac{t_\nu^G}{t_\nu} = \frac{\lambda_G}{\lambda_C}\left[1 - (2\omega R \sin\alpha)^2/c^2\right]^{-1} = \gamma^2\frac{\lambda_G}{\lambda_C}, \quad (13)$$

where $\gamma \geq 1$ is the usual relativity factor as defined above.

Now, λ_G/λ_C is simply the ratio between the spatial displacement between our consecutive gedanken control pulses, being these displacements defined through our previous paragraphs, leading to the Eqs. (4) and (10). Of course, this ratio is simply given by the relativistic Doppler effect under an approximation case in which C_C is the source and C_G the detector. The ratio between the Eqs. (10) and (4) gives:

$$\frac{\lambda_G}{\lambda_C} = [1 - (2\omega R \sin\alpha)/c]\frac{\left(t_\nu^G - \delta t_\nu^G\right)}{(t_\nu - \delta t_\nu)}. \quad (14)$$

But the time interval $(t_\nu - \delta t_\nu)$ is a proper time interval measured by the source clock C_C, as previously discussed. It accounts for the time interval between the first pulse sent and the last pulse sent as locally described by C_C is its $O_C x_C y_C z_C \equiv Oxyz$ world. These two events accur at different spatial locations in the C_G detector clock world $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$, since C_C is approximating to C_G is this latter world. Hence, $t_\nu - \delta t_\nu$ is the Lorentz time contraction of $t_\nu^G - \delta t_\nu^G$, viz.:

$$t_\nu - \delta t_\nu = \gamma^{-1}\left(t_\nu^G - \delta t_\nu^G\right) \quad \therefore$$

$$\frac{\left(t_\nu^G - \delta t_\nu^G\right)}{t_\nu - \delta t_\nu} = \gamma = \left[1 - (2\omega R \sin\alpha)^2/c^2\right]^{-1/2}. \quad (15)$$

With the Eqs. (14) and (15), one reaches the usual relativistic Doppler effect expression for the approximation case:

$$\frac{\lambda_G}{\lambda_C} = \sqrt{\frac{1 - (2\omega R \sin\alpha)/c}{1 + (2\omega R \sin\alpha)/c}}. \quad (16)$$

With the Eq. (16), the Eq. (13) reads:

$$\frac{t_\nu^G}{t_\nu} = \left[1 - (2\omega R \sin\alpha)^2/c^2\right]^{-1/2}[1 + (2\omega R \sin\alpha)/c]^{-1} =$$

$$= \frac{\gamma}{1 + (2\omega R \sin\alpha)/c}. \quad (17)$$

Since $(2\omega R \sin\alpha)/c << 1$, we may apply an approximation for the Eq. (17), viz.:

$$\gamma \approx 1 + \frac{1}{2}\frac{(2\omega R \sin\alpha)^2}{c^2}, \quad (18)$$

and:

$$[1 + (2\omega R \sin\alpha)/c]^{-1} \approx 1 - (2\omega R \sin\alpha)/c, \quad (19)$$

from which, neglecting the higher order terms, the Eq. (17) reads:

$$\frac{t_\nu^G}{t_\nu} \approx 1 - \frac{2\omega R \sin\alpha}{c} \quad \therefore \quad (20)$$

$$t_\nu^G - t_\nu = -\frac{2\omega R \sin\alpha}{c}t_\nu. \quad (21)$$

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From this result, the clock that tag the arrival interaction instant t_ν^G in Gran Sasso turns out to measure an arrival time that is shorter than the correct one, this latter given by t_ν. With the discrepancy, ϵ, given by the value measured by the OPERA Collaboration [1], since t_ν is simply given by L/v_ν, where L is the baseline distance between the CERN and Gran Sasso, v_ν the speed of neutrino through the Earth crust, one obtains a value for v_ν. We rewrite the Eq. (21):

$$\epsilon = t_\nu^G - t_\nu = -\frac{2\omega R \sin\alpha}{c}\frac{L}{v_\nu}. \tag{22}$$

With the values* $\omega = 7.3\times10^{-5}s^{-1}$, $R = R_E\cos\lambda \approx 6.4\times10^6 m\times\cos(\pi/4) = 4.5\times10^6 m$, $\sin\alpha \approx \sin(7\pi/180) = 1.2\times10^{-1}$, $c = 3.0\times10^8 ms^{-1}$ and $L = 7.3\times10^5 m$, also with the discrepancy ϵ, given by the Eq. (22), being, say, $\epsilon = -62\times10^{-9}s$, the neutrino velocity through the Earth crust reads:

$$v_\nu \approx 3.1\times10^6\, ms^{-1}, \tag{23}$$

being the refraction index of the Earth crust for neutrino given by:

$$n_{c|\nu} = \frac{c}{v_\nu} \approx 97. \tag{24}$$

In reference to the matching PDF (probability distribution function) in the OPERA experiment, one would have a discrepancy between the maximum likelihood distribution obtained from the block of detectors at Gran Sasso and the translation of the PDF due to the protons distribution by TOF_c given by, in virtue of the Eq. (22):

$$\mathrm{TOF}_\nu = \mathrm{TOF}_c + \epsilon = \mathrm{TOF}_c - \frac{2\omega R\sin\alpha}{c}\frac{L}{v_\nu} \therefore$$

$$\mathrm{TOF}_\nu - \mathrm{TOF}_c \approx -62\, ns, \tag{25}$$

under the reasoning and simplifications throughout this paper. One should notice the resoning here holds if the discrepancy turns out to be encrusted within the time translation of the PDF data, but such effect would not arise if the time interval TOF_ν were directly measured, since, in this latter situation, such interval would only read L/v_ν.

Conclusion

It is interesting to observe that even with a velocity having got two orders of magnitude lesser than c a neutrino may be interpreted as having got a velocity greater than c, depending on the method used to measure neutrino's time of flight, with the Earth crust presenting a index of refraction $n_{c|\nu} > 1$, due, also, to the local Doppler effect between the clocks attached to Gran Sasso and the respective intersecting ones in the CERN reference frame, as discussed throughout this paper, due to the Earth daily rotation.

*See the Eqs. (2) and (3). The latitudes of CERN and Gran Sasso are, respectively: $46^{\text{deg}}14^{\text{min}}3^{\text{sec}}$(N) and $42^{\text{deg}}28^{\text{min}}12^{\text{sec}}$(N). The longitudes of CERN and Gran Sasso are, respectively: $6^{\text{deg}}3^{\text{min}}19^{\text{sec}}$(E) and $13^{\text{deg}}33^{\text{min}}0^{\text{sec}}$(E).

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References

1. The OPERA collaboration: T. Adam et al. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam http://arxiv.org/abs/1109.4897 *arXiv:1109.4897*, 2011.
2. Armando V.D.B. Assis. Prog. Phys. 4(4), 85 to 90 (2011).

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