Energy, Momentum, Mass and Velocity of Moving Body

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In the weak-field approximation the problem of 4/3 is formulated for internal and external gravitational fields of a body in the form of a ball. The dependence of the energy and mass of the moving substance on energy of field accompanying the substance, as well as dependence on the characteristic size of the volume occupied by substance are found. Additives in the energy and momentum of the body, defined by energy and momentum of the gravitational and electromagnetic fields associated with the body are explicitly defined. The conclusion is that energy and mass of the body can be described by the energy of usual and strong gravitation, and through the energy of electromagnetic fields of particles that compose the body.

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In relativistic mechanics, there are standard formulas for the total energy and momentum of a particle:

$$E=\frac{mc^2}{\sqrt{1-V^2/c^2}}, \qquad \boldsymbol{p}=\frac{m\boldsymbol{V}}{\sqrt{1-V^2/c^2}}, \tag{1}$$

where V is velocity of the particle with mass m.

If the energy E and momentum $\boldsymbol{p}$ in (1) is known the mass and velocity of the particle may be calculated:

$$m=\frac{1}{c^2}\sqrt{E^2-p^2c^2}, \qquad \boldsymbol{V}=\frac{\boldsymbol{p}c^2}{E}. \tag{2}$$

In (1) and (2) is the speed of light c. For a particle in rest velocity and momentum are zero, and the total energy of the particle is the rest energy:

$$E_0=mc^2. \tag{3}$$

Equation (3) reflects the principle of proportionality of mass and energy. In elementary particle physics energy and momentum are usually measured parameters, and mass and velocity are found from (2) and are secondary parameters.

Now, suppose that the measured parameters are the energy and velocity of the particle. In this case, from (1) we can calculate the mass and momentum:

$$m = \frac{E\sqrt{1-V^2/c^2}}{c^2}, \qquad \mathbf{p} = \frac{E\mathbf{V}}{c^2}. \tag{4}$$

It is also possible the case when the measured parameters are the momentum and velocity of the particle and calculated quantities are the mass and energy:

$$m = \frac{p\sqrt{1-V^2/c^2}}{V}, \qquad E = \frac{p\,c^2}{V}. \tag{5}$$

If the particle velocity V is given, then the mass can be found either through the energy according to (4), or through the momentum according to (5), in both cases, the mass should be the same.

There are also two possible combinations of parameters when the energy and mass are known, or momentum and mass. This let us to calculate the magnitude of momentum and velocity, or energy and velocity, respectively:

$$p = \frac{1}{c}\sqrt{E^2 - m^2c^4}, \qquad V = \frac{c}{E}\sqrt{E^2 - m^2c^4},$$

$$E = c\sqrt{p^2 + m^2c^2}, \qquad \mathbf{V} = \frac{\mathbf{p}\,c}{\sqrt{p^2 + m^2c^2}}.$$

From the above formulas is not clear whether they contain in themselves the energy and momentum of fields, which are inherent in the particles and the test bodies. In particular, the test bodies always have their own gravitational field and can still carry an electrical charge and the corresponding electromagnetic field. The main purpose of this paper is to incorporate explicitly in the relativistic formulas for the energy and momentum of the additives, resulting from the energy and momentum of fields associated with the test bodies.

External gravitational field. Problem of 4/3

Assume that (1) – (5) are valid for a small particle and its mass take into account the energy of own gravitational field. If there are a lot of particles in the volume of the body, then their interaction energy leads to a significant contribution of the field energy in total energy of the body. In order to simplify calculations, we will further consider the case of a weak field. This means that any spacetime metric is little different from the metric of Minkowski spacetime, or the gravitational effects of time dilation and size reducing considerably smaller than similar effects due to body movement. In this approximation the general theory of relativity becomes gravitomagnetism, the covariant theory of gravitation reduces to Lorentz-invariant theory of gravitation (LITG) [1], and the field equations in gravitomagnetism and in LITG are the same.

Gravitational potentials of element of substance, located at $t=0$ in a point (x_0, y_0, z_0) of space and moving along axis OX at a constant speed V, according to [2] are as follows:

$$d\psi = -\frac{\gamma dM}{\sqrt{1-V^2/c^2}\sqrt{\frac{(x-x_0-Vt)^2}{1-V^2/c^2}+(y-y_0)^2+(z-z_0)^2}}, \qquad d\boldsymbol{D} = \frac{d\psi\,\boldsymbol{V}}{c_g^2}, \qquad (6)$$

here $d\psi$ – scalar potential,

γ – gravitational constant,

dM – mass of element of substance,

c_g – propagation speed of gravity, which further to simplify the calculations, we assume equal to the speed of light c,

(x, y, z) – coordinates of the point at which the potential is determined at the time t,

$d\boldsymbol{D}$ – vector potential.

According to (6), the gravitational potential $d\psi$ at the time t of the point mass dM as it moves along the axis OX depends on the initial position (x_0, y_0, z_0) of the mass at $t=0$. After integration of (6) over all point masses inside a ball based on the principle of superposition, the standard formulas for the potentials of gravitational field around moving ball, with retardation of the gravitational interaction taken into account, are obtained:

$$\psi = -\frac{\gamma M}{\sqrt{(x-Vt)^2+(1-V^2/c^2)(y^2+z^2)}}, \qquad \boldsymbol{D} = \frac{\psi \boldsymbol{V}}{c^2}, \qquad (7)$$

where ψ – scalar potential of a moving ball,

M – mass of the ball,

(x, y, z) – coordinates of the point at which the potential is determined at the time t (with the condition that the center of the ball was at $t=0$ in the origin of coordinate system),

$\boldsymbol{D}$ – vector potential.

In (7) assumed that the ball is moving along axis OX at a constant speed V, so that $D_x = \frac{\psi V}{c^2}$, $D_y = 0$, $D_z = 0$. With the help of field potentials we can calculate field strengths around the ball by the formulas [3]:

$$\boldsymbol{G} = -\nabla\psi - \frac{\partial \boldsymbol{D}}{\partial t}, \qquad \boldsymbol{\Omega} = \nabla \times \boldsymbol{D}, \tag{8}$$

where $\boldsymbol{G}$ is the gravitational acceleration,

$\boldsymbol{\Omega}$ – gravitational torsion in LITG (gravitomagnetic field in gravitomagnetism).

In view of (7) and (8) we find:

$$G_x = -\frac{\gamma M\left(x - Vt\right)(1 - V^2/c^2)}{\sqrt{[\left(x-Vt\right)^2 + (1-V^2/c^2)(y^2+z^2)]^3}}, \qquad G_y = -\frac{\gamma M\, y\,(1 - V^2/c^2)}{\sqrt{[\left(x-Vt\right)^2 + (1-V^2/c^2)(y^2+z^2)]^3}},$$

$$G_z = -\frac{\gamma M\, z\,(1 - V^2/c^2)}{\sqrt{[\left(x-Vt\right)^2 + (1-V^2/c^2)(y^2+z^2)]^3}}, \qquad \Omega_x = 0, \tag{9}$$

$$\Omega_y = \frac{\gamma M\, z\, V(1 - V^2/c^2)}{c^2\sqrt{[\left(x-Vt\right)^2 + (1-V^2/c^2)(y^2+z^2)]^3}}, \quad \Omega_z = -\frac{\gamma M\, y\, V(1 - V^2/c^2)}{c^2\sqrt{[\left(x-Vt\right)^2 + (1-V^2/c^2)(y^2+z^2)]^3}}.$$

The energy density of the gravitational field is determined by the formula:

$$u = -\frac{1}{8\pi\gamma}\left(G^2 + c^2\Omega^2\right) = -\frac{\gamma M^2(1-V^2/c^2)[\left(x-Vt\right)^2 + (1+V^2/c^2)(y^2+z^2)]}{8\pi\,[\left(x-Vt\right)^2 + (1-V^2/c^2)(y^2+z^2)]^3}. \tag{10}$$

The total energy of the field outside the ball at a constant velocity should not depend on time. So it is possible to integrate the energy density (10) over the external space volume at $t = 0$. For this purpose we introduce new coordinates:

$$x = \sqrt{1-V^2/c^2}\; r\cos\theta, \qquad y = r\sin\theta\cos\varphi, \qquad z = r\sin\theta\sin\varphi. \tag{11}$$

Volume element is determined by the formula $d\Upsilon = J\, dr\, d\theta\, d\varphi$, where:

$$J=\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial\theta} & \frac{\partial x}{\partial\varphi}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta} & \frac{\partial y}{\partial\varphi}\\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial\theta} & \frac{\partial z}{\partial\varphi}\end{vmatrix}.$$

It follows that $d\Upsilon=r^2\sin\theta\sqrt{1-V^2/c^2}\,dr\,d\theta\,d\varphi$. Integral over the space of the energy density (10) will be:

$$U_b=\int u\,d\Upsilon=-\frac{\gamma M^2}{8\pi c^2\sqrt{1-V^2/c^2}}\int\frac{[c^2+V^2(\sin^2\theta-\cos^2\theta)]\sin\theta\,dr\,d\theta\,d\varphi}{r^2}. \qquad (12)$$

We note that due to the Lorentz reduction in the motion along the axis OX the ball must be as an ellipsoid, the surface equation of which at $t=0$ is such:

$$\frac{x^2}{1-V^2/c^2}+y^2+z^2=R^2. \qquad (13)$$

After substituting (11) in (13), it becomes apparent that the radius r of the integration in (12) must change from R to ∞, and the angles θ and φ change the same way as in spherical coordinates (between 0 and π for the angle θ, and from 0 to 2π for the angle φ). For the energy of the gravitational field outside of a moving ball we find:

$$U_b=-\frac{\gamma M^2(1+V^2/3c^2)}{2R\sqrt{1-V^2/c^2}}=\frac{U_{b0}(1+V^2/3c^2)}{\sqrt{1-V^2/c^2}}, \qquad (14)$$

where $U_{b0}=-\frac{\gamma M^2}{2R}$ is field energy around of stationary ball.

Assume that the formula (4) for connection of mass and energy of a particle is also valid for the gravitational field. Then we can introduce the effective mass of the field related to energy:

$$m_{gb}=\frac{U_b\sqrt{1-V^2/c^2}}{c^2}=\frac{U_{b0}(1+V^2/3c^2)}{c^2}. \qquad (15)$$

We now consider the momentum density of the gravitational field:

$$\boldsymbol{g} = \frac{\boldsymbol{S}}{c^2}, \qquad (16)$$

where $\boldsymbol{S} = -\frac{c^2}{4\pi\gamma}[\boldsymbol{G}\times\boldsymbol{\Omega}]$ is the vector of energy flux of the gravitational field.

Substituting in (16) components of the field (9), we find:

$$g_x = -\frac{\gamma M^2 (1-V^2/c^2)^2 (y^2+z^2) V}{4\pi c^2 \, [(x-Vt)^2 + (1-V^2/c^2)(y^2+z^2)]^3}, \qquad (17)$$

$$g_y = \frac{\gamma M^2 (1-V^2/c^2)^2 (x-Vt)\, y\, V}{4\pi c^2 \, [(x-Vt)^2 + (1-V^2/c^2)(y^2+z^2)]^3},$$

$$g_z = \frac{\gamma M^2 (1-V^2/c^2)^2 (x-Vt)\, z\, V}{4\pi c^2 \, [(x-Vt)^2 + (1-V^2/c^2)(y^2+z^2)]^3}.$$

It is seen that the components of the momentum density of gravitational field (17) look the same as if a liquid flowed around the ball from the axis OX, carrying a similar density of momentum – liquid spreads out to the sides when meeting with the ball and merges once again on the opposite side of the ball. Integrating the components of the momentum density of gravitational field (17) by volume outside of moving ball at $t=0$ as in (12), we obtain:

$$\Gamma_x = \int g_x \, d\Upsilon = -\frac{\gamma M^2 V}{4\pi c^2 \sqrt{1-V^2/c^2}} \int \frac{\sin^3\theta \, dr\, d\theta\, d\varphi}{r^2} = -\frac{2\gamma M^2 V}{3Rc^2\sqrt{1-V^2/c^2}}. \qquad (18)$$

$$\Gamma_y = \int g_y \, d\Upsilon = 0, \qquad \Gamma_z = \int g_z \, d\Upsilon = 0.$$

In (18) the total momentum of the field has only the component along the axis OX. By analogy with (5) the coefficient before velocity V in (18) can be interpreted as the effective mass of the external gravitational field moving with the ball:

$$m_{pb} = \frac{\Gamma_x \sqrt{1-V^2/c^2}}{V} = -\frac{2\gamma M^2}{3Rc^2} = \frac{4U_{b0}}{3c^2}, \qquad (19)$$

where $U_{b0}=-\frac{\gamma M^2}{2R}$ is the energy of the external static field of the ball in rest.

Comparing (19) and (15) gives:

$$m_{gb}=\frac{3(1+V^2/3c^2)\,m_{pb}}{4}. \qquad (20)$$

The discrepancy between the masses m_{gb} and m_{pb} in (20) is the essence of the so-called problem of 4/3, according to which the mass m_{pb} of the field, which is calculated through the momentum of the field, at low speeds approximately 4/3 larger than the mass m_{gb} of the field, found through the field energy. A characteristic feature of the fundamental fields, which include the gravitational and electromagnetic fields, is the similarity of their equations for the potentials and field strengths. The problem of 4/3 is known for a long time for the mass of electromagnetic field of a moving charge. Joseph John Thomson, George Francis FitzGerald, Oliver Heaviside [4], George Frederick Charles Searle and many others wrote about it in the late 19-th century. We also discussed this question previously with respect to the gravitational field of a moving ball [5]. Now we present an exact solution of the problem, not limited to the approximation of small velocities.

The gravitational field inside a moving ball

According to [2] for a ball with the density of substance $\bar{\rho}$ (measured in comoving frame), which is moving along the axis OX, the potentials inside the ball (denoted by subscript i) depend on time and are as follows:

$$\psi_i=-\frac{2\pi\gamma\bar{\rho}}{\sqrt{1-V^2/c^2}}\left[R^2-\frac{1}{3}\left(\frac{(x-Vt)^2}{1-V^2/c^2}+y^2+z^2\right)\right], \qquad \boldsymbol{D}_i=\frac{\psi_i\mathbf{V}}{c^2}. \qquad (21)$$

In view of (8) we can calculate strengths of internal field:

$$G_{xi}=-\frac{4\pi\gamma\bar{\rho}\left(x-Vt\right)}{3\sqrt{1-V^2/c^2}}, \qquad G_{yi}=-\frac{4\pi\gamma\bar{\rho}\,y}{3\sqrt{1-V^2/c^2}}, \qquad G_{zi}=-\frac{4\pi\gamma\bar{\rho}\,z}{3\sqrt{1-V^2/c^2}},$$

$$\Omega_{xi}=0, \qquad \Omega_{yi}=\frac{4\pi\gamma\bar{\rho}\,zV}{3c^2\sqrt{1-V^2/c^2}}, \qquad \Omega_{zi}=-\frac{4\pi\gamma\bar{\rho}\,yV}{3c^2\sqrt{1-V^2/c^2}}. \qquad (22)$$

Similarly to (10) for the energy density of the field we find:

$$u_i = -\frac{1}{8\pi\gamma}\left(G^2 + c^2\Omega^2\right) = -\frac{2\pi\gamma\bar{\rho}^2[(x-Vt)^2 + (1+V^2/c^2)(y^2+z^2)]}{9(1-V^2/c^2)}. \qquad (23)$$

According to (23) the maximum energy density inside a moving ball is achieved on its surface, and in the center at $t=0$ is zero.

The integral of (23) by volume of the ball at $t=0$ in coordinates (11) with the volume element $d\Upsilon = r^2 \sin\theta \sqrt{1-V^2/c^2}\, dr\, d\theta\, d\varphi$ yields:

$$U_i = \int u_i\, d\Upsilon = -\frac{2\pi\gamma\bar{\rho}^2}{9c^2\sqrt{1-V^2/c^2}} \int [c^2 + V^2(\sin^2\theta - \cos^2\theta)] r^4 \sin\theta\, dr\, d\theta\, d\varphi. \qquad (24)$$

The moving ball looks like an ellipsoid with surface equation (13), and in the coordinates (11) radius of the integration in (24) varies from 0 to R. With this in mind for the energy of the gravitational field inside the moving ball, we have:

$$U_i = -\frac{\gamma M^2(1+V^2/3c^2)}{10R\sqrt{1-V^2/c^2}} = \frac{U_{i0}(1+V^2/3c^2)}{\sqrt{1-V^2/c^2}}, \qquad (25)$$

where $U_{i0} = -\frac{\gamma M^2}{10R}$ is the field energy inside a stationary ball of radius R.

Effective mass of the field may be associated with energy and is obtained similarly (4):

$$m_{gi} = \frac{U_i\sqrt{1-V^2/c^2}}{c^2} = \frac{U_{i0}(1+V^2/3c^2)}{c^2}. \qquad (26)$$

Substituting in (16) components of the field strengths (22), we find the components of the vector of momentum density of gravitational field:

$$g_{xi} = -\frac{4\pi\gamma\bar{\rho}^2(y^2+z^2)V}{9c^2(1-V^2/c^2)}, \qquad g_{yi} = \frac{4\pi\gamma\bar{\rho}^2(x-Vt)\,yV}{9c^2(1-V^2/c^2)}, \qquad g_{zi} = \frac{4\pi\gamma\bar{\rho}^2(x-Vt)\,zV}{9c^2(1-V^2/c^2)}. \qquad (27)$$

Vector connecting the origin of coordinate system and center of the ball depends on the time and has the components $(Vt, 0, 0)$. From this in the element of substance, coinciding with the center of the ball, the momentum density of gravitational field is always zero. At $t=0$ the center of the ball passes through the origin of coordinate system, and at the time from (27) follows that the maximum density of the field momentum $g_{\max}=-\frac{4\pi\gamma\bar{\rho}^2 R^2 V}{9c^2(1-V^2/c^2)}=-\frac{\gamma M^2 V}{4\pi R^4 c^2(1-V^2/c^2)}$ is achieved on the surface of the ball on the circle of radius R in the plane YOZ, which is perpendicular to the line OX of ball motion. The same follows from (17).

We can integrate the components of the momentum density of gravitational field (27) over the volume inside a moving ball at $t=0$ with the coordinates (11) similar to (24):

$$\Gamma_{xi}=\int g_{xi}\,d\Upsilon=-\frac{4\pi\gamma\bar{\rho}^2 V}{9c^2\sqrt{1-V^2/c^2}}\int r^4\sin^3\theta\,dr\,d\theta\,d\varphi=-\frac{2\gamma M^2 V}{15Rc^2\sqrt{1-V^2/c^2}}. \qquad (28)$$

$$\Gamma_{yi}=\int g_{yi}\,d\Upsilon=0, \qquad \Gamma_{zi}=\int g_{zi}\,d\Upsilon=0.$$

As in (18), the total momentum of field (28) has only the component along the axis OX. By analogy with (5) the coefficient before velocity V in (28) is interpreted as the effective mass of gravitational field inside of the ball:

$$m_{pi}=\frac{\Gamma_{xi}\sqrt{1-V^2/c^2}}{V}=-\frac{2\gamma M^2}{15Rc^2}=\frac{4U_{i0}}{3c^2}, \qquad (29)$$

where $U_{i0}=-\frac{\gamma M^2}{10R}$ is the field energy within a stationary ball.

Comparing (26) and (29) gives:

$$m_{gi}=\frac{3(1+V^2/3c^2)\,m_{pi}}{4}. \qquad (30)$$

Connection (30) between the masses of the field inside the ball is the same as in (20) for the masses of the external field, so the problem of 4/3 is inside the ball too.

The contribution of gravitational field in energy and momentum of a moving body

Let's try to include in equation (1) the relations found above for the energy and momentum of the gravitational field of moving test body in the form of a ball. Suppose that in static case instead of (3) there is the next relation:

$$E_0 = E_{0s} - U_0\,, \tag{31}$$

where E_{0s} – the rest energy of substance,

$U_0 = U_{b0} + U_{i0} = -\frac{3\gamma M^2}{5R}$ – total energy of static gravitational field inside and outside the ball with uniform density of the substance.

Choosing the minus sign before U_0 in (31) will be validated in the last section. We will continue to analyze the well-known thought experiment. Assume that the substance of the ball is composed of matter and antimatter, which at some time begin to annihilate and emit photons. Suppose that photons fly in opposite directions along the axis OX of the number of N in each direction, so that eventually the entire mass of the ball turns into electromagnetic radiation. In the course of emission because of equality of all momentums of photons, and symmetry of the radiation along the axis OX the ball remains stationary. In order that the process does not depend on the radius of the ball, we put the radius is a constant irrespective of changes in mass. The energy E_0 of the ball (31) should be transformed into the energy of photons:

$$E_0 = 2Nh\nu_0\,, \tag{32}$$

where h – Planck constant,

ν_0 – frequency of photons.

Consider the same situation in the frame of reference K, in which the ball moves with constant velocity V along the axis OX and at $t = 0$ is in the origin of coordinate system. We believe that the speed of the ball does not change, despite the emission of photons. In the frame K frequency of the photons will depend on whether they are flying along the axis OX or in the opposite direction. Taking into account the relativistic Doppler effect and (31), for photon energy instead of (32) will be:

$$\begin{gathered} E = Nh\nu_1 + Nh\nu_2 = \frac{Nh\nu_0\sqrt{1-V^2/c^2}}{1-V/c} + \frac{Nh\nu_0\sqrt{1-V^2/c^2}}{1+V/c} = \\ = \frac{2Nh\nu_0}{\sqrt{1-V^2/c^2}} = \frac{E_0}{\sqrt{1-V^2/c^2}} = \frac{E_{0s} - U_0}{\sqrt{1-V^2/c^2}}. \end{gathered} \tag{33}$$

On the other hand, the total energy of the gravitational field inside and outside of the ball, taking into account (14) and (25) is:

$$U = U_b + U_i = -\frac{3\gamma M^2 (1+V^2/3c^2)}{5R\sqrt{1-V^2/c^2}} = \frac{U_0\,(1+V^2/3c^2)}{\sqrt{1-V^2/c^2}}\,. \qquad (34)$$

For the energy of substance and field of the moving ball, we have:

$$E = E_s - U = E_s - \frac{U_0\,(1+V^2/3c^2)}{\sqrt{1-V^2/c^2}}\,. \qquad (35)$$

From (33) and (35):

$$E_s = \frac{E_{0s}}{\sqrt{1-V^2/c^2}} + \frac{U_0\,V^2}{3c^2\sqrt{1-V^2/c^2}}\,. \qquad (36)$$

Since the energy of static field is negative: $U_0 = -\frac{3\gamma M^2}{5R}$, then in (36) energy E_s of substance of moving ball will be reduced by the addition of field energy.

Consider now the law of conservation of momentum. Before emission of the photons momentum of the moving ball consist of substance momentum and momentum of the gravitational field, and taking into account (18) for the field momentum outside the ball, and (28) for the momentum of the field inside the ball, the total momentum of the field is:

$$P_f = -\frac{4\gamma M^2\,V}{5Rc^2\sqrt{1-V^2/c^2}}\,.$$

Then for the momentum of a moving ball can be written:

$$P = P_s + P_f = M_V V - \frac{4\gamma M^2\,V}{5Rc^2\sqrt{1-V^2/c^2}} = M_V V + \frac{4U_0\,V}{3c^2\sqrt{1-V^2/c^2}}\,, \qquad (37)$$

where M_V is the mass of substance of the ball as a function of velocity V.

After photon emission the whole momentum of the ball and its gravitational field becomes equal to the momentum of photons:

$$P=\frac{Nh\nu_1}{c}-\frac{Nh\nu_2}{c}=\frac{2Nh\nu_0 V}{c^2\sqrt{1-V^2/c^2}}=\frac{E_0 V}{c^2\sqrt{1-V^2/c^2}}=\frac{(E_{0s}-U_0)V}{c^2\sqrt{1-V^2/c^2}}, \qquad (38)$$

where $E_0=E_{0s}-U_0$ is the energy (31) of the ball in rest, equal to the difference of the rest energy E_{0s} of substance and energy of the gravitational field U_0; also E_0 is the energy of photons according to (32). By comparing (37) and (38):

$$E_{0s}=M_V c^2\sqrt{1-V^2/c^2}+\frac{7U_0}{3}. \qquad (39)$$

Suppose that the mass of the moving substance of the ball is described by the formula: $M_V=\frac{\alpha M}{\sqrt{1-V^2/c^2}}$, where M – the mass of substance at rest, and α is a function. Here we assume that the mass M of substance at rest, and that the mass of the substance through which is determined the energy U_0 and momentum P_f of the gravitational field, is the same mass. Then instead of (39) will be:

$$E_{0s}=\alpha M c^2+\frac{7U_0}{3}. \qquad (40)$$

But the energy E_{0s} of substance at rest should not depend on speed, as well as by (31) on the field energy U_0 of a stationary ball. Therefore, in (40) should be $\alpha=1-\frac{7U_0}{3Mc^2}$, which implies the following:

$$E_{0s}=Mc^2, \qquad M_V=\frac{M}{\sqrt{1-V^2/c^2}}-\frac{7U_0}{3c^2\sqrt{1-V^2/c^2}}. \qquad (41)$$

Substitute E_{0s} from (41) in (36):

$$E_s = \frac{Mc^2}{\sqrt{1-V^2/c^2}} + \frac{U_0 V^2}{3c^2\sqrt{1-V^2/c^2}}. \qquad (42)$$

In (42) at $V = 0$ the energy of the substance does not include field energy, but when driving in the energy E_s of the substance appears additive, related with energy U_0 of field. The field energy U_0 also makes contribution to the mass M_V of the moving substance in (41). The total energy (35) of moving substance and field in view of (42) will be:

$$E = \frac{Mc^2 - U_0}{\sqrt{1-V^2/c^2}}, \qquad (43)$$

where in the case of a uniform density of the ball substance $U_0 = -\frac{3\gamma M^2}{5R}$.

Equation (43) implies that the energy of the body increases due to the contribution of negative gravitational energy U_0.

We now substitute M_V from (41) in (37) or E_{0s} in (38). This gives the following:

$$\boldsymbol{P} = \frac{(Mc^2 - U_0)\boldsymbol{V}}{c^2\sqrt{1-V^2/c^2}}. \qquad (44)$$

By comparing (43) and (44) with (1) shows that taking into account of gravitational field role of the total mass of the substance and the field plays quantity $M_\Sigma = M - U_0/c^2$. If we know the energy E (43) and momentum $\boldsymbol{P}$ (44), it follows from these relations that we can express the mass M of the substance and speed V of the body (if mass and size dependence of the gravitational energy U_0 of the body is known). In the case of a homogeneous ball of radius R in the calculation of the mass M of the moving substance of the ball need to solve a quadratic equation, the result will be:

$$M = -\frac{5Rc^2}{6\gamma}\left(1 - \sqrt{1 + \frac{12\gamma\sqrt{E^2 - p^2c^2}}{5Rc^4}}\right), \qquad \boldsymbol{V} = \frac{\boldsymbol{P}c^2}{E}. \qquad (45)$$

According to (45), the mass of the body substance depends not only on the energy-momentum of the body, but also on the average body size due to the contribution of the mass of the gravitational field.

Note also that the problem of 4/3 for the gravitational field (inequality mass of the field, found from the energy, and mass of the field, calculated by the momentum of the field) was compensated in dependence of energy E_s in (36) and mass M_V in (41) of moving substance from the field energy U_0. As a result, the field energy U_0 in formulas (43) and (44) is symmetrical in both total energy and total momentum of the body.

Analysis of components of mass and energy of a body

Until now we did not specify from which components consist the mass M of the substance of a body, is there a contribution of other energies except the energy of gravitational field? For example, what happens when the body heat? From the standpoint of kinetic theory, an increase in temperature leads primarily to an increase in the average velocity of the particles that make-up the body. In this case, according to (1) the average energy of each particle of the body increases, and by the additivity of energy should be changed the total energy E_Σ of the body at rest. For the case of substance and gravitational field $E_\Sigma = Mc^2 - U_0$, and (43) – (44) can be written as:

$$E = \frac{E_\Sigma}{\sqrt{1-V^2/c^2}}, \qquad \mathbf{P} = \frac{E_\Sigma \mathbf{V}}{c^2\sqrt{1-V^2/c^2}}. \tag{46}$$

Heating the body leads to a change of E_Σ in (46), and heat as a form of energy is distributed between the rest energy Mc^2 of substance and energy U_0 of gravitational field. Mass of uniform ball can be determined from the relations:

$$E_\Sigma = Mc^2 - U_0 = Mc^2 + \frac{3\gamma M^2}{5R}, \qquad M = -\frac{5Rc^2}{6\gamma}\left(1-\sqrt{1+\frac{12\gamma E_\Sigma}{5Rc^4}}\right). \tag{47}$$

Any interaction between particles of a body itself or with environment, which changes the energy of the particles, changes the energy E_Σ of the body at rest. In accordance with (47) mass M of substance of the ball depends not only on E_Σ, but also on the radius of the ball R.

Due to the similarity of equations of electromagnetic and gravitational fields, the energy E_Σ must contain contribution from the total energy W_0 of the electromagnetic field of the body:

$$E_\Sigma = M c^2 - U_0 - W_0 . \tag{48}$$

For uniformly charged by volume ball in rest with a charge q the total energy of the electric field is:

$$W_0 = \frac{3q^2}{20\pi\varepsilon_0 R} .$$

Magnetic field can produce contribution to the energy W_0 , if the ball is magnetized or if there are electric currents. We assume that other forms of energy (e.g. heat) can change the body mass, but can not change the charge of the body, because it needs to be transferred to the body (from the body) charged particles. This is one of the differences between the electromagnetic and gravitational fields, in addition to the unipolarity of gravitational charges (which are masses) and the bipolarity of electromagnetic charges.

Body mass M in (48) can be divided into two parts, one of which M_g is the body mass at zero temperature in Kelvin, and the other part M_T is the additional mass from the internal thermal energy E_T, which includes the kinetic energy of chaotic motion of atoms and molecules, and energy of turbulent motion of substance fluxes. If V_T is an average thermal velocity of particles in the body, then there are the following approximate relations: $E_T \approx \frac{M V_T^2}{2}$, $M_T = \frac{E_T}{c^2} \approx \frac{M V_T^2}{2c^2}$. As the energy of fields, we include the energy E_T in (48) with negative sign:

$$E_\Sigma = M_g c^2 - E_T - U_0 - W_0 . \tag{49}$$

For bodies that are only under its own gravitational and electromagnetic fields, the virial theorem is satisfied, according to which the modulus of potential energy of the field on average is twice as much internal heat:

$$2E_T + U_0 + W_0 \approx 0 , \qquad E_{tot} = E_T + U_0 + W_0 \approx -E_T \approx \frac{U_0 + W_0}{2} , \tag{50}$$

here E_{tot} is total energy excluding rest energy of particles of the body.

Substituting (50) in (49) gives the approximate equality:

$$E_{\Sigma} = M_g c^2 - E_{tot} \approx M_g c^2 - \frac{U_0 + W_0}{2} . \qquad (51)$$

We now consider the essence of mass M_g related to body mass excluding the contribution from the mass of heat and macroscopic fields. There are some contribution in mass M_g from the masses of various types of energy associated with atoms and molecules near absolute zero: strong interaction, fastening the substance of the elementary particles and retaining the nucleons in atomic nuclei; electromagnetic interaction of particles; the energy of motion of electrons in atoms; rotational energy of atoms and molecules; vibrational energy of atoms in molecules, etc. In Standard Model assumes that the strong interaction arises due to the action of the gluon field between the quarks located in the hadrons (mesons and baryons), and the strong interaction between leptons is absent.

There is also a hypothesis that the strong interaction is a manifestation of strong gravitation at the level of elementary particles and atoms [6]. Since gravitation has two components, as the field of acceleration G and the torsion field Ω, the stability of nucleons in nuclei can be described as the balance of force from the attraction of the nucleons to each other through G, and repulsion of nucleons by the torsion field Ω [1]. The same idea is applied to describe the structure and stability of a number of hadrons, considered as the composition of the nucleons and mesons [2]. Strong gravitation different from the usual gravitation by replacing the gravitational constant γ on the constant of strong gravitation Γ, and acts between all particles, including leptons. Estimation of the quantity Γ can be obtained from the balance of four forces acting on an electron in a hydrogen atom: 1. The force of electric attraction between electron and atomic nucleus. 2. The force of electric repulsion of the charged substance of electron itself from itself (the electron is represented as a cloud around the nucleus). 3. Centripetal force on the rotation of electron around the nucleus. 4. The attraction of electron to the nucleus under the influence of strong gravitation. These forces are approximately equal to each other, so there are the relation for the forces of attraction of strong gravitation and electric force:

$$-\frac{\Gamma M_p M_e}{R_e^2} = -\frac{e^2}{4\pi\varepsilon_0 R_e^2} , \qquad \Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29} \ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{c}^{-2} , \qquad (52)$$

where M_p and M_e – mass of proton and electron, respectively,

R_e – radius of rotation of electron cloud,

e – elementary electric charge as proton charge and modulus of negative charge of electron,

ε_0 – vacuum permittivity.

Another way to estimate Γ based on the theory of similarity of matter levels and the use of coefficients of similarity. These factors are defined as follows: $\Phi = 1.62 \cdot 10^{57}$ – coefficient of similarity on mass (the ratio of mass of neutron star to proton mass); $P = 1.4 \cdot 10^{19}$ – coefficient of similarity on size (the ratio of radius of neutron star to proton radius); $S = 0.23$ – coefficient of similarity on velocity (the ratio of characteristic velocity of particles of neutron star to the speed of light as a typical velocity of proton substance). For strong gravitational constant take place a formula: $\Gamma = \gamma \frac{\Phi}{PS^2}$, where powers of similarity coefficients correspond to dimension of gravitational constant according to the dimensional analysis.

If understand the strong interaction as a result of strong gravitation, the main contribution to the proton rest energy should make positive internal energy of its substance and negative energy of the strong gravitation (the electrical energy of the proton can be neglected because of its smallness). The sum of these energies gives the total energy of the proton, and by virtue of the virial theorem (50) this sum of energies is approximately equal to half the energy of strong gravitation. Since the energy of the strong gravitation is negative, then the total energy of the proton is negative too. The total energy of the proton can be regarded as binding energy of its substance; modulus of binding energy equal to the work that needs to do to spread the substance to infinity so that there total energy of substance (potential and kinetic) was equal to zero. According to its meaning, positive proton rest energy must be equal to absolute value of total energy (modulus of binding energy) of proton. This gives the equality between the rest energy and the modulus of half the energy of strong gravitation:

$$M_p c^2 = \frac{k \Gamma M_p^2}{2 R_p}, \tag{53}$$

where $k = 0.6$ for the case if the proton was homogeneous density ball of radius R_p.

If in (53) to substitute (52), we get another equation, which allows estimating the radius of proton:

$$M_e c^2 = \frac{k e^2}{8 \pi \varepsilon_0 R_p}, \qquad R_p = \frac{k e^2}{8 \pi \varepsilon_0 M_e c^2} = \frac{k r_0}{2},$$

where r_0 is classical electron radius.

In self-consistent model of proton [2] is found that in (53) radius of proton $R_p = 8.73 \cdot 10^{-16}$ m, and the coefficient $k = 0.62$ due to a small increase in density of substance in the center of proton. At the same time, under the assumption that positive charge distributed over volume of proton similar to mass distribution and the maximum angular velocity of proton is limited by the condition of its integrity in the field of strong gravitation, there is the magnetic moment of proton as a result of rotation of charged substance:

$$P_m = \delta e \sqrt{\Gamma M_p R_p} \ , \tag{54}$$

where $P_m = 1.41 \cdot 10^{-26}$ J/T is the magnetic moment of proton,

$\delta = 0.1875$ (in the case of uniform density and charge of proton should be $\delta = 0.2$).

Strong gravitational constant (52) explains not only the energy (53) and magnetic moment (54) of proton, but also gives an estimate of constant of interaction between two nucleons by the strong gravitation:

$$\alpha_{pp} = \frac{\beta \Gamma M_p^2}{\hbar c} = 13.4\,\beta$$

where $\beta = 0.26$ for the interaction of two nucleons, and tends to 1 for particles with lower density of substance. Constant α_{pp} is close to coupling constant α_s of strong interaction of two nucleons in Standard Model, in which $\alpha_s \approx 14.6$.

The fact that the rest energy of proton is associated with strong gravitation, it also follows from the modernized Fatio-Le Sage theory of gravitation [7]. In this theory, based on the absorption of the flux of gravitons in substance of bodies with a momentum transfer of gravitons to substance an exact formula for Newton's gravitational force (inverse square law) is derived; the energy density of the flux of gravitons ($4 \cdot 10^{34}$ J/m^3), the section of their interaction with substance ($7 \cdot 10^{-50}$ m^2) and other parameters are deduced.

In theory of infinite hierarchical nesting of matter [1], [3] is shown that at each main level of matter appears corresponding type of gravitation: there is a strong gravitation at the level of elementary particles, but at the level of stars – the usual gravitation. Gravitation reaches a maximum in the densest objects – in nucleons and in neutron stars. In substance of the earth's density range of strong gravitation is less than a meter, and at such sizes of bodies strong gravitation is replaced by the usual gravitation. This corresponds to the fact that the masses and sizes of objects at different levels of matter increase exponentially, and the point of replacing of strong gravitation by the usual gravitation lies near the middle of the range of masses from nucleons to the stars on the axis of the mass in logarithmic scale.

As the cause of gravitation and electric forces are considered fluxes of gravitons, which consist of particles similar to neutrinos, photons and charged particles. These fluxes of gravitons generated by substance of lower levels of matter, control bodies with the help of gravitational and electromagnetic forces and create massive objects at higher levels of matter. These objects, in turn, at certain stages of their evolution radiate portions of neutrinos, photons and charged particles that become the basis of other fluxes of gravitons, are already operating at higher levels of matter. So the field and massive objects mutually generate each other at different levels of matter.

In the above picture the rest energy of proton (53) is approximately equal to the modulus of the total energy of the proton in its own field of strong gravitation (for increased accuracy should also take into account the electromagnetic energy of the proton), and the energy $M_g c^2$ in (51) consists of the rest energy of nucleons and electrons of substance of a body, with the addition of energy of their gravitational and electromagnetic interactions inside the substance and mechanical motion in atoms and molecules. Consequently, the energy $M_g c^2$ of the body, taking into account the virial theorem (50) can be reduced to half of the modulus of the sum energy of strong gravitation U_{0g} and electromagnetic energy W_{0g} of the nucleons, electrons, atoms and molecules involved in formation of the binding energy. As a result, the total energy of substance and fields of a body at rest instead of (51) can be written:

$$E_\Sigma \approx -\frac{U_{0g}+W_{0g}}{2}-\frac{U_0+W_0}{2}. \qquad (55)$$

To better understand the meaning of energy E_Σ, we consider the energy balance in a process of merging of substance, with formation of elementary particles at the beginning, passing then to confluence of the elementary particles into atoms and finally to formation of a body of many atoms. Initially, the substance is motionless at infinity and its parts do not interact with each other, so that energy of the system is zero. If the substance will draw together under the influence of gravitation, it appears the negative energy U of gravitational field and the positive kinetic energy E_T of motion of substance, and in view of energy conservation the total energy should not change, remaining equal to zero. In the energy balance is necessary to take into account the electromagnetic energy W and the energy E_r of field quanta such as photons and neutrinos leaving the system due to the radiation:

$$E_r + E_T + U + W = 0, \qquad E_r = -(E_T + U + W) = -E_{Tot} = -\frac{U+W}{2}. \qquad (56)$$

In (56) is used the virial theorem (50) for the components of the total energy E_{Tot} of the system. According to (56), the energy E_r of radiation leaving the system up to a sign equals to the total energy E_{Tot}, i.e. energy E_r equals the binding energy of the system, taken with a minus sign. By comparing (56) and (55) we now show that the energy E_Σ of substance and field of a body at rest is the same as the energy extracted from the body by different radiation during formation of the body. Currently, the energy E_Σ is accounted for only those components that are associated with formation of elementary particles, atoms and macroscopic molecular substance; and energy of the particles of which substance of elementary particles themselves is built are not counted and assumed to be constant. Heating the body in accordance with (56) and (55) leads to an increase in body energy E_Σ. This conclusion stems from the fact that although the internal thermal energy of a body E_T is a part of (56) with a negative sign, but the change of potential energy $U+W$ from the virial theorem compensates the contribution of the energy E_T.

According to (55), the total energy E_Σ of a body at rest, which is used in formulas (46) for calculation of energy and momentum of the moving body, composed mainly of the energies of two fundamental fields – gravitational and electromagnetic, as responsible for the integrity of the particles of the body and for the composition of the body of the individual particles. In this case, the strong interaction between the particles is taken into account by the energy of strong gravitation U_{0g} and electromagnetic energy W_{0g}.

In the picture is assumed that the weak interaction is the result of transformation of substance, which is for a long time under the influence of the fundamental fields. An example is the long-term evolution of star massive enough to form a neutron star into a supernova explosion, when the neutrino burst is emitted with an energy about total energy of the star (the energy of gravitational collapse of substance in small-sized neutron star is converted into energy of neutrinos, radiation energy and kinetic energy and heating of the threw shell). At the level of elementary particles, this corresponds to the process of formation of a neutron with the emission of neutrino.

If from a body at rest in the weak interaction are emitted (absorbed by the body) neutrinos, photons and other particles, it leads to a change in the total energy E_Σ of the body. In general, the energy E_Σ of the body is a function of time and the speed with which the individual particles or elements of substances emitted from the body or absorbed them. By virtue of the laws of conservation of energy and momentum, if some particles are brought into the system the energy and momentum, then after some time they are distributed in the system and according to virial theorem can be accounted for through the energy and momentum of the fundamental fields. Therefore, it can be argued that according to (55), the source of the total energy of the body, and its mass M_Σ as a measure of inertia are the gravitational and electromagnetic fields associated with the mass and charge (as well as currents) in the substance. In Fatio-Le Sage theory of gravitation supposed that the fields associated with the mass and charges are a consequence of the interaction of substance and charges with the fluxes of gravitons and tiny charged particles that penetrate the space. If we define the total mass of the body in the form $M_\Sigma = \frac{E_\Sigma}{c^2}$, then (46) is as follows:

$$E = \frac{M_\Sigma c^2}{\sqrt{1-V^2/c^2}}, \qquad \boldsymbol{P} = \frac{M_\Sigma \boldsymbol{V}}{\sqrt{1-V^2/c^2}}. \tag{57}$$

Equations (57) look exactly the same as (1) for a small test particle. However, body mass M_Σ in (57) takes full account of the field energy, whereas for the mass of small particles in (1) it is only expected. The appearance in the mass M_Σ of contribution from the energy of fields has occurred because we have used the energy of mutual interaction of many small particles in massive body. Hence, by induction, we should suppose that not only mass of body, but the mass of any isolated small particle should be determined taking into account the contribution from the energy of own fundamental fields of the particle.

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