Advances in Black Body Radiation
(A Source of the Cosmological Constant)

By

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Abstract: The past century has seen many dividends from Planck's theory for black body radiation. Today we advance that theory to provide an explanation for the acceleration of the Universe and other cosmological observations. We do this by arguing:

- Thermal diffusion of an *individual* massive particle will build up excess energy until it releases that energy in the form of black body radiation.
- Virtual particle pairs also suffer from such a mechanism and if an atom is nearby to capture the radiation before the pair annihilates, a net negative energy is pulled from the vacuum.
- During the Dark Ages of the Universe when hydrogen atoms were prevalent, these virtual particles were coupled to and in thermal equilibrium with the background radiation field (today known as the Cosmic Microwave Background, CMB).
- As the Universe re-ionized at $z \approx 10$, the virtual particles decoupled from the CMB, leaving the temperature of the virtual particles and the associated energy density to stay constant as the Universe continued to expand.
- When the magnitude of this constant density is inserted into the Friedmann equation, it is interpreted as the Cosmological Constant predicting, as observed, an accelerating Universe.
- Using the 68% confidence ranges as measured from the 5 year Wilkinson Microwave Anisotropy Probe, the magnitude of the Cosmological Constant as calculated from the theory described herein overlaps the value from the well accepted Lambda Cold Dark Matter model.

Lastly with the intent to build support for further investigation, conclusions are presented which include: proposals for experimental verification, application to other cosmological observations and a rationalization based on Occam's razor.

Background and Approach: With recent cosmological observations that the Universe is not only expanding but accelerating, focus has fallen on an elusive energy density with negative pressure coined "Dark Energy" that has an equation of state, $w < -1/3$ (Liddle 2003). Previous attempts to explain its origin however have fallen short (Greene 2004; Peebles & Ratra 2003). Yet that has not kept many theories from being put forward (Papantonopoulos 2007); with the most accepted being the Lambda Cold Dark Matter model, ΛCDM (Liddle 2003). Here Lambda, the Cosmological Constant, is a constant energy density with negative pressure, $w=-1$.

In this article, the author proposes that the Cosmological Constant is indeed the correct form of dark energy and furthermore that black body radiation is the source of that energy. The approach of this argument will be to give a simple explanation of how and why virtual particles are the source of a constant energy density that has the effect of accelerating the expansion of space. The argument rests on a mechanism by which virtual particles radiate like a black body, which is described in detail in the appendix. We will call this model the Lambda Black Body Radiation Model, ΛBBR

Black Body Radiation: A Black Body is an object that absorbs any impingent radiation. Such a body will radiate a spectrum of power dependent only on its temperature as given by (Reif 1965),

$$\wp(\omega)d\omega = \frac{\hbar\omega^3}{4\pi^2c^2}\frac{1}{\left(e^{\hbar\omega/_{kT}} - 1\right)}d\omega$$

The energy density (kg/m^3) of this distribution is calculated by integrating the black body spectrum over the frequency and solid angle and transforming the units (Reif 1965).

$$\rho_{BBR} = \frac{1}{c^3}\int_0^\infty \wp(\omega)d\omega \int d\Omega = \frac{\pi^2(kT)^4}{15\hbar^3c^5}$$

While it is commonly understood that black body radiation is associated with macroscopic objects, the appendix shows that the black body spectrum is fundamental to an *individual* particle.

Process for Virtual Particles Pairs: Virtual particles should too suffer from this mechanism. For a virtual particle pair, the radiation process begins with the pair coming into existence. They begin diffusing and accumulating excess energy; after a given time, the excess energy is released. The two particles then collapse back on each other. If a sink is not around to absorb the radiation from the virtual particles before the pair re-annihilates, the emitted photon will not escape, leaving no evidence of the pair. If on the other hand an atom or other sink is nearby that has the right transition frequency to collect the radiation before the pair re-annihilates, the emitted radiation will provide a real positive energy to the Universe. In order to keep the conservation of energy, a net negative energy must remain in the vacuum with the same magnitude but opposite sign of energy density as black body radiation.

$$\rho_{\text{Virtual Particles}} = -\rho_{BBR}$$

It is not all too surprising that a mechanism which traps energy from virtual particles leaves evidence in the form of black body radiation, since we have seen something similar in the Hawking Radiation from black holes (Hartle & Hawking 1976). In Hawking Radiation, one of the two virtual particles gets trapped inside the event horizon of a black hole. Of the pair, the free one escapes as black body radiation while the trapped one causes the black hole to lose mass. In this case, it is not the massive particles that get trapped, but rather excess energy built up from the pair's diffusive motion. However the result is the same; black body radiation is emitted and a negative energy remains to balance out the conservation of energy.

If this mechanism is the only means by which virtual particles can exchange heat, their temperature will track the local radiation field (which for most of the Universe is the background radiation) assuming enough atoms are present to capture emitted photons. This was the situation in the Universe during what is known as the Dark Ages; the time between decoupling and re-ionization (Barkana & Loeb 2001). However once re-ionization occurred, the atoms, which facilitated the equilibrium between virtual particles and the background radiation field, were lost. At this time virtual particles stopped cooling and kept a constant temperature even unto today leaving a negative energy density, $\rho_{\text{Virtual Particles}}$.

$$\rho_{\text{Virtual Particles}} = -\frac{\pi^2 (kT_\Lambda)^4}{15\hbar^3 c^5}$$

Given the approximate red-shift of re-ionization this final temperature of the virtual particles, T_Λ, can be calculated (Peacock 1999).

$$T_\Lambda = (1 + z_{Re-ionization})T_{today} = constant$$

As long at the virtual particles did not continue to cool but rather stayed at the temperature of re-ionization, T_Λ, the Universe was left with a constant and pervasive dark energy density.

Constant Density: Right after the end of the Dark Ages this dark energy density was the same magnitude as the radiation density of the observable Universe. However while the temperature of the virtual particles remained constant, along with its associated density, the temperature of the background radiation continued to cool and its associated radiation density scaled like one over the fourth power of the scale factor. As we know, this radiation density of the observable Universe is called today the Comic Microwave Background.

There is interesting physics that takes place here as it relates to conservation of energy. In the case of the radiation density and non-relativistic energy, the total energy (density times volume) remains constant in the case of non-relativistic matter, or decreases like one over the scale factor in the case of radiation since the wavelength of the radiation are themselves expanding (Peacock 1999). However the case is different for a constant energy density. We can see this by looking at the equation of state (Reif 1965).

$$\bar{p}_{\text{Virtual Particles}} = \sum_s \bar{n}_s \left(-\frac{\partial E_s}{\partial V}\right) = -\frac{\partial}{\partial V}\sum_s \bar{n}_s E_s = -\frac{\partial}{\partial V}(\rho_{\Lambda BBR} \cdot V) = -\rho_{\text{Virtual Particles}}$$

In the case of an equation of state of w=-1, the total energy (density times volume) actually increases linearly with volume. For a reversible process in the presence of an ideal gas we have (Reif 1965),

$$dE = -dW = -pdV$$

$$dE_{\text{Virtual Particles}} = \rho_{\text{Virtual Particles}} \cdot dV$$

In other words, as the Universe expands a real amount of positive work (constant pressure times volume) causes the total energy of the virtual particles to be in negative direct proportion with the volume thus keeping density constant (Liddle 2003).

As time passed both the radiation and matter density decreased while the density from the virtual particles stayed constant. At some point the magnitude of the virtual particle energy density became greater than both the radiation and non-relativistic matter density making the constant energy density (with its negative pressure) the dominant gravitational force pushing the Universe apart at an accelerating rate.

The Cosmological Constant: Before the Cosmological Constant, Λ, is introduced, we first make the assumption of *normal* gravity (Nieto & Goldman 1991; Karshenboim 2008), i.e. that all energy (even negative energy) attracts all other energy (whether positive or negative). With this assumption the effective energy density is

$$\rho_{\Lambda\mathrm{BBR}} = |\rho_{\mathrm{Virtual\ \ Particles}}| = \frac{\pi^2 (kT_\Lambda)^4}{15\hbar^3 c^5}$$

However it can be shown that even under the assumption of *normal* gravity, the effects of a constant density with negative pressure can be repulsive at astronomical scales (Mannheim 2000).

We see this by introducing the scale factor of the Universe, R(t) (Peacock 1999),

$$ds^2 = R(t)^2 \cdot ds_3^2 - c^2 dt^2$$

As shown by Friedmann, R(t) is related to the Hubble Constant, H (Liddle 2003),

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho(R) - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}$$

As we can see, the introduction of Λ is simply a replacement for a scaled version of $\rho_{\Lambda\mathrm{BBR}}$

$$\Lambda = \frac{8\pi G}{c^2}\rho_{\Lambda BBR} = \frac{8\pi^3 G (kT_\Lambda)^4}{15\hbar^3 c^7} = constant$$

Λ is a function of Universal constants and T_Λ, which has been argued is also constant; leading to an equation of state of w=-1. As we see in the next section where we estimate the magnitude of $\rho_{\Lambda BBR}$, Λ dominates the Freidmann equation. Ignoring other factors of $\rho(R)$ that are a function of R(t) and the contribution from curvature, we are left with,

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{\Lambda c^2}{3}} = constant$$

Which when solved predicts (as observed) that the Universe is not only expanding but accelerating

$$R(t) = R_o e^{Ht}$$

$$\ddot{R}(t) = H^2 R(t) > 0$$

Agreement with ΛCDM: Because we have estimates of today's z value of re-ionization and today's temperature of the CMB we can estimate the density $\rho_{\Lambda BBR}$ using the Lambda Black Body Radiation model, ΛBBR.

Without going into the details, the Lambda Cold Dark Matter model, $\Lambda\mathrm{CDM}$, provides a completely independent estimate of the density of dark energy, $\rho_{\Lambda\mathrm{CDM}}$ (Liddle 2003).

$$\rho_{\Lambda CDM} = \Omega_\Lambda \cdot \rho_{\text{critical}} = \Omega_\Lambda \cdot \frac{3H^2}{8\pi G}$$

Again we can estimate $\rho_{\Lambda\text{CDM}}$ because we have estimates of the parameter, Ω_Λ, and today's Hubble constant, H.

The source of our estimates will be from the 5 year Wilkinson Microwave Anisotropy Probe (Dunkley et al., 2009). However we must keep in mind that these estimates are best fit parameters and come with confidence ranges. The ranges and estimates are compiled below in Table 1 and visualized in Figure 1.

	low	average	high		low	average	high
$\rho_{\Lambda BBR}$ (kg/m^3)	5.86E-27	1.03E-26	1.50E-26	$\rho_{\Lambda CDM}$ (kg/m^3)	6.41E-27	7.40E-27	8.05E-27
T_{today} $(degrees)$	2.725	2.725	2.725	H $\left(\frac{km}{sec \cdot Mpc}\right)$	69.2	72.4	74.5
$z_{re-ionization}$	9.6	11.2	12.4	Ω_Λ	0.712	0.751	0.772

Table 1

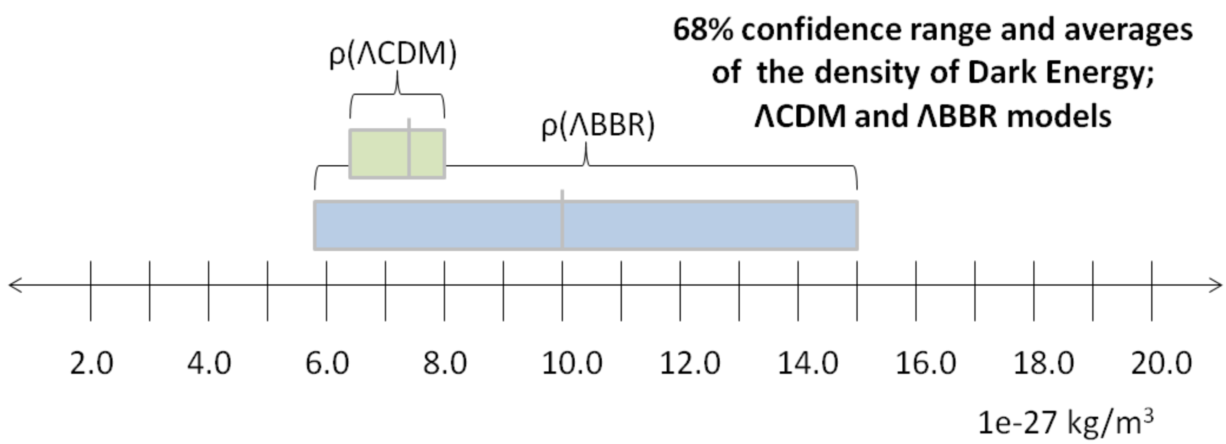

Figure 1

Three things are of note. First, the ranges of the density of dark energy from the two models overlap implying good agreement. Second, ΛBBR reliance on the fourth power of the z value of re-ionization and its own confidence range causes the 68% confidence range for $\rho_{\Lambda BBR}$ to be four times as broad as the range for $\rho_{\Lambda CDM}$. Perhaps further modeling of re-ionization and its impact on the ΛBBR model could reduce its confidence range improving the likelihood of overlap. Third, the range of $\rho_{\Lambda CDM}$ lies on the lower side of the range of $\rho_{\Lambda BBR}$. This last note can be accounted for if the temperature of the virtual particles T_Λ continue to slightly cool (due to a few atoms remaining) after re-ionization. Again further modeling of the ΛBBR model would bring insight and precision to these estimates.

Conclusion: The Lambda Black Body Radiation model described herein is based on the black body radiation of virtual particles, and provides an explanation of dark energy in the form of the Cosmological Constant. Thus the evidence of dark energy we are looking for has already been found; it is the Cosmic Microwave Background. The only thing missing in our search was that the dark energy density that balanced the CMB stopped cooling and remained constant as the epic of the Dark Ages ended.

One might ask, "How can we prove such a model is correct?" Outlined in the appendix is an experiment that would go a long way to prove that an *individual* particle can radiate like a Black Body. It uses the result from the appendix that an *individual* particle diffuses classically on top of other motion and should be visible in the measurement of a quantum walk (Karski et al. 2009) (assuming the temperature and time scales are in the right range).

Another experiment might be to measure the temperature of virtual particles by isolating a sufficiently high density of hydrogen atoms. If the hydrogen atoms tend to approach the temperature T_Λ, it could indcate the hydrogen atoms are coupled to the virtual particles and maintaining equilibrium. However it is not clear on what time scales this would happen.

Alternatively, one might look for cosmological observations where the environment is such that the virtual particles are coupled to the baryonic matter. The case of VIRGOHI21 (Minchin et al. 2005) is such a region where there are plenty of neutral hydrogen atoms to facilitate equilibrium of the virtual particles. In this local region the environment might be quite similar to that of the Universe during the Dark Ages; as such the equation of state of the virtual particles could be positive and their temperature would be in equilibrium with the local temperature. In this case the resulting energy density of the virtual particles due to black body radiation is still be negative, but would have positive pressure resulting in attractive gravity. If this is correct, it could explain the very large proportion of observed dark matter; where the dark matter is really the result of a combination of positive pressure exerted from the virtual particles (that are in equilibrium with the local temperature) on top of the absence of the otherwise ubiquitous negative pressure.

The opposite environment is insightful too. For example Globular Clusters provide a region of the Universe where starlight is very prevalent. In this case we would expect any atoms, or other loose matter to be fully ionized and thus not able to provide the coupling mechanism to allow the local virtual particles to cool. In this example $\rho_{\Lambda BBR}$ would remain constant, the equation of state would be w = -1 and we would not see the effects of any additional positive pressure. If this reasoning is correct, it could explain why we don't measure any dark matter within Globular Clusters (Mashchenko & Sills 2005).

These two observational examples provide anecdotal evidence that ΛBBR not only provides an explanation for dark energy, but for dark matter as well. While this would be a very satisfying solution (tying together two open questions of cosmology), more investigation is needed.

Additional questions that remain include (but are not limited to):

1) Are there other mechanisms by which virtual particles can couple to ordinary energy?
2) What is the rate of heat transfer between virtual particles and ordinary energy as a function of the density of neutral atoms?

3) Can we model how virtual particles cool in the presence of ordinary matter in order to explain the observed density of dark matter?
4) What are the impacts to entropy?
5) How does ΛBBR hold up to Quantum Mechanics?
6) How does ΛBBR hold up to General Relativity?

Further questions about the radiation process are also given in the appendix.

New physics is needed to further understand how an *individual* particle (and for that matter virtual particles) radiate like a Black Body. Still, the ΛBBR model (where a virtual particle radiates like a black body and couples to the background radiation field) is quite simple and complements our current understanding for the observed acceleration of the Universe. With the ΛBBR model being more simple and accurate than other attempts (Papantonopoulos 2007) to reconcile the reason for acceleration, the author calls upon Occam's razor (Cover & Thomas 1991) to argue for further investigation of these findings.

Appendix: Derivation of the Black Body Spectrum for an *Individual* massive particle

Appendix - Hypothesis: When Planck solved the black body spectrum by using an assumption of quantized energy levels in 1900, it was unlikely that he had in mind the idea that space and time are also quantized (Ranganath 2008). Even in 1922 when Stern and Gerlach experimentally showed "directional" quantization, the explanation was that a particle's angular momentum was quantized, not that a particle could only step in discrete quanta (Mehra & Rechenberg 1999; Feynman 1965). However after a century of advances in physics, electrical engineering and computer science (where discrete time processing has taken on a fundamental role (Shannon 1949; Forsythe & Wasow 1960; Bracewell 1986)), such an assumption has an opportunity to gain traction.

Taking it a step further, this analysis hypothesizes that on top of all other motion a massive particle steps in space in discrete quanta and diffuses in space via the discrete Bernoulli process. Specifically the location of the center of a particle's wavepacket in each of the three position dimensions, will each independently step in the positive or negative direction an amount dx, dy, and dz at each time step dt with probability p or (1-p) respectively; moving its location from x, to x+dx or x-dx depending on a Boolean random event. For the current analysis we assume that p=½ and thereby use a reference frame that is moving with zero average velocity relative to the particle.

- It is hypothesized that the primary mechanism causing the black body radiation of an *individual* particle is that "on top of all other motion, an individual particle diffuses through space by stepping a discrete quantum via the Bernoulli process."

Appendix - The process and its parameters: The first step is to derive the parameters of the Bernoulli process as they relate to the appropriate physical quantities. Details of the Bernoulli process can be reviewed in either Chandrasakhar (1942) or Reif (1965).

First, the time step dt is found through the Heisenberg uncertainty principle where $E \cdot dt = \hbar/2$ (Shankar 1994). With $E=mc^2$ (Einstein 1956) we arrive at

$$dt = \frac{\hbar}{2E} = \frac{\hbar}{2mc^2}$$

The total time will be t=K·dt, where K is the number of steps in the process.

The position step, dx, dy, and dz will be equal through symmetry and equal to the speed of light times dt. We arrive at this conclusion since the velocity eigenvalue of the Dirac equation for a massive free particle is equal to the speed of light, c (Dirac 1958). The particle does not violate special relativity because the total displacement x (which will have a Binomial distribution) will rapidly become much less the total time t as the number of steps increases, thus preserving a group velocity which is less than c. One can think of the particle moving the distance dx in time dt by turning into pure energy of value mc^2, yet stopping at each time step to possibly turn around.

$$dx = dy = dz = c \cdot dt = \frac{\hbar}{2mc}$$

We can now calculate the distribution on x as the two sided binomial distribution with probability parameter ½ and step size dx. We calculate the variance Δx^2 as

$$(\Delta x)^2 = dx^2 \cdot K = \frac{K\hbar^2}{4m^2c^2}$$

If K is large, we can approximate the Binomial distribution as the Gaussian distribution

$$x \sim p(x) = \frac{1}{\sqrt{2\pi\Delta x^2}} \cdot e^{-\frac{x^2}{2\Delta x^2}}$$

We can now calculate the distribution on the momentum p_x, p_y, and p_z. The distribution on p_x can also be approximated by a Gaussian with a variance that satisfies the Heisenberg uncertainty principle (Shankar 1994).

$$(\Delta p_x)^2 = \frac{\hbar^2}{4(\Delta x)^2} = \frac{m^2c^2}{K}$$

$$p_x \sim p(p_x) = \frac{1}{\sqrt{2\pi\Delta p_x{}^2}} \cdot e^{-\frac{p_x{}^2}{2\Delta p_x{}^2}}$$

Thus the probability distribution on x and p_x (with y & z and p_y & p_z being respectively equal), will be approximated by the Gaussian distribution with mean zero and variance Δx^2 and $\Delta p_x{}^2$ respectively. However care needs to be taken when calculating the diffusion constant for this classical diffusive term as there is reciprocity between the position and the momentum. Thus when calculating the diffusion constant we must account for variance from both the randomness in position and the displacement from the randomness in momentum. Calling this total variance $(\Delta x)^2_{Classical}$ we can derive the diffusion constant, D.

$$(\Delta x)^2_{Classical} = 2Dt = (\Delta x)^2 + \frac{(\Delta p_x)^2}{m^2}t^2$$

$$(\Delta x)^2_{Classical} = dx^2 \cdot K + \frac{m^2c^2}{K}\frac{1}{m^2}(dt \cdot K)^2 = 2dx \cdot ct$$

From which we get,

$$D = \frac{\hbar}{2m}$$

Appendix - A Way for Experimental Verification: It should be noted that this classical diffusion associated with the Bernoulli process should not be confused with the quantum mechanical diffusion of a free particle's wave packet. Due to phase interactions of the pure frequencies that make up a free particle's wave packet in the momentum space, quantum mechanical diffusion of the wave packet in the position space (also known as a quantum walk (Karski et al. 2009)) is derived by solving the imaginary diffusion equation (kinetic energy Hamiltonian) and has a characteristic variance that is quadratic in time (Shankar 1994).

$$(\Delta x)^2_{QM} = (\Delta x_0)^2 + \frac{k_B T}{m} t^2 = (\Delta x_0)^2 + \frac{(\Delta p_0)^2}{m^2} t^2$$

This spreading of the wavepacket is independent of the classical diffusion of the center of the wavepacket that occurs due to the process described herein. To calculate the variance in position that would be observed if a measurement were to occur t seconds after a free particle at temperature T was initialized in a minimum uncertainty state, we would add the quantum variance to the classical variance because the two are derived from independent Gaussian distributions.

$$(\Delta x)^2_{Observed} = (\Delta x)^2_{QM} + (\Delta x)^2_{Classical}$$

One way to provide support in affirmation of our given hypothesis is to make a measurement of the variance of a free particle. As indicated above, a measurement of a free particle t seconds after initialization in the minimum uncertainty state will be $(\Delta x)^2_{Observed}$

$$(\Delta x)^2_{Observed} = (\Delta x_0)^2 + \frac{(\Delta p_0)^2}{m^2} t^2 + \frac{\hbar}{m} t = \left((\Delta x_0) + \frac{(\Delta p_0)}{m} t\right)^2$$

Looking at Figure (2) of $(\Delta x)^2_{Observed}$ *and* $(\Delta x)^2_{QM}$ over a normal time frame shows little divergence and experiments of quantum mechanical diffusion could easily have missed the extra classical term in the noise of the measurements (Karski et al. 2009). However in the range of $\frac{\hbar}{4k_B T} < t < \frac{\hbar}{k_B T}$, $(\Delta x)^2_{Classical}$ is the dominant term and $(\Delta x)^2_{Observed}$ is measurably different from $(\Delta x)^2_{QM}$.

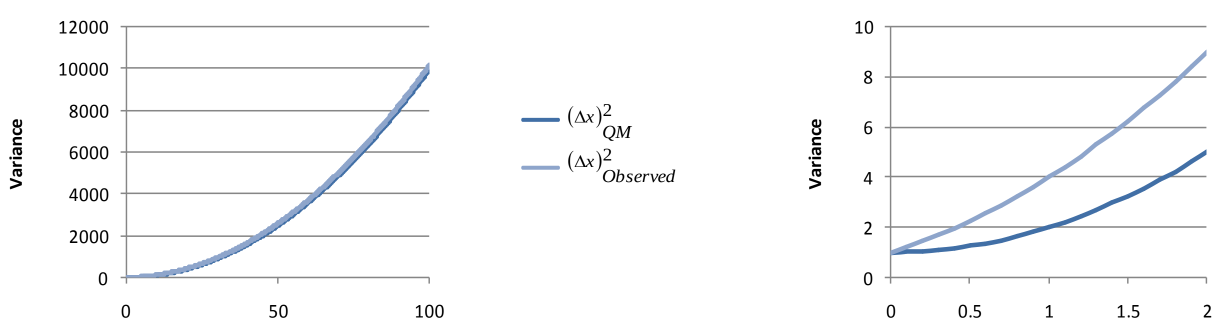

Figure 2 – Hypothesized solution $(\Delta x)^2_{Observed}$ vs. textbook solution $(\Delta x)^2_{QM}$ for normal and short time scales

Luckily at 10mK, the temperature of a dilution refrigerator, the width of an electron's wavepacket at initialization is 150nm. This length scale is large enough to detect with current state of the art semiconductor manufacturing. A measurement of a free particle at this temperature and a few nanoseconds after initialization should show the effects of the classical Bernoulli diffusion.

Appendix - Black Body Radiation: The idea that an individual particle radiates with a probability distribution that is in agreement with the black body spectrum has already been shown for faster than light particles / tachyons (Musha 2002). However as can be seen below our argument is not limited to faster than light particles, but is a consequence of our hypothesis for all massive particles. That is the probability distribution of the power of emitted radiation per unit area per solid angle in the frequency range $[\omega, \omega + d\omega]$ by a single thermal particle fits perfectly to the proven solution (Reif 1965)

$$\wp(\omega)d\omega = \frac{\hbar\omega^3}{4\pi^2c^3}\frac{1}{\left(e^{\hbar\omega/kT} - 1\right)}d\omega$$

The model is simple: a particle starts out at rest; it diffuses via the Bernoulli process for K steps; it comes to rest again and radiates its excess energy in the form of photons.

We begin by calculating the available energy of a particle after K steps. Traditionally the kinetic energy is proportional to the quadratic sum over the three momentum directions. However similar to the reciprocity described above when calculating the diffusion constant, the motion of the particle in the x, y and z directions give the particle an associated group velocity x/t, y/t, and z/t bringing the total available energy to be the quadratic sum of the six dimensions.

$$Available\ Energy = \hbar\omega = \hbar\omega_x + \hbar\omega_{px} + \hbar\omega_y + \hbar\omega_{py} + \hbar\omega_z + \hbar\omega_{pz}$$

$$Available\ Energy = \hbar\omega = \tfrac{m}{2}\left(\tfrac{x}{t}\right)^2 + \tfrac{1}{2m}(p_x)^2 + \tfrac{m}{2}\left(\tfrac{y}{t}\right)^2 + \tfrac{1}{2m}\left(p_y\right)^2 + \tfrac{m}{2}\left(\tfrac{z}{t}\right)^2 + \tfrac{1}{2m}(p_z)^2$$

After K steps, the particle will be found in phase space in each of the three dual dimensions: (x,p_x); (y,p_y); and (z,p_z) as shown in Figure (3). Over either an ensemble of particles, or over time (assuming the particle has an energy source that keeps it from cooling after it radiates), the distribution in phase space will be Gaussian. The equivalent energy lines are ellipses from which we can calculate the distribution on the total available energy, ω.

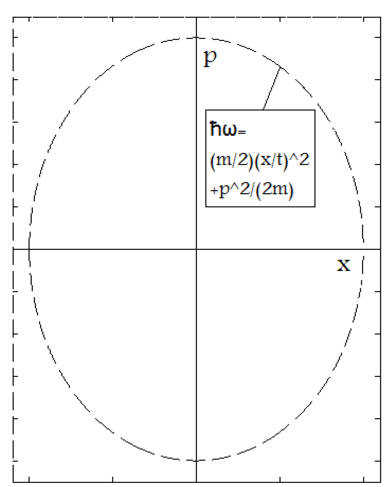

Figure 3 – Available energy in phase space

Given the relationship $\omega_x = ax^2$ (a>0) and with x~p(x) as above, the distribution on ω_x is (Pan 2007)

$$\omega_x \sim p(\omega_x) = \frac{1}{\sqrt{2\pi\Delta x^2 a\omega_x}} e^{\frac{-\omega_x}{2a\Delta x^2}}$$

With this relationship and the probability distributions on x, y, z, p_x, p_y, and p_z given above we derive ω_x

$$\omega_x \sim p(\omega_x) = \sqrt{\frac{K\hbar}{\pi mc^2 \omega_x}} e^{\frac{-K\hbar\omega_x}{mc^2}}$$

Fortunately the distribution on ω_{px} is the same

$$\omega_{px} \sim p(\omega_{px}) = \sqrt{\frac{K\hbar}{\pi mc^2 \omega_{px}}} e^{\frac{-K\hbar\omega_{px}}{mc^2}}$$

Since ω is simply the sum of the independent variables ω_x, ω_y, ω_z and ω_{px}, ω_{py}, ω_{pz}, we can solve for the distribution on ω as the convolution of the other distributions (Bracewell 1986).

$$\omega \sim p(\omega) = p(\omega_x) * p(\omega_y) * p(\omega_z) * p(\omega_{px}) * p(\omega_{py}) * p(\omega_{pz})$$

$$\omega \sim p(\omega) d\omega = \frac{K^3 \hbar^3 \omega^2}{2m^3 c^6} e^{\frac{-K\hbar\omega}{mc^2}} d\omega$$

To go from $p(\omega)d\omega$ to $\wp(\omega)d\omega$, we must normalize by the total surface area will be $4\pi R^2 = 4\pi(K\cdot c\cdot dt)^2$; and the solid angle of 4π since the photons will radiate away from the particle radially at the speed of light. Also to go from the probability distribution on photons to the power spectrum we must multiply by the power per photon $\hbar\omega/t$ where t = Kdt.

$$\wp_K(\omega)d\omega = \frac{\hbar\omega \cdot p(\omega)d\omega}{(4\pi(K\cdot c\cdot dt)^2)\cdot(4\pi)\cdot(Kdt)} = \frac{\hbar\omega^3}{4\pi^2 c^2} e^{\frac{-K\hbar\omega}{mc^2}} d\omega$$

We are almost there, but two more steps are needed. In our assumption, the excess energy will be radiated by a photon of energy ħω; as we know the energy of the photons is quantized and there can actually be M photons per mode with total energy M·ħω. If we assume that the process is ergodic and that the particle has been in isolation since it was in thermal equilibrium with a reservoir at temperature T, we can enlist the help of the equipartition theorem to solve for M (Feynman 1965). Doing so, we see the average energy is equal to $k_BT/2$ for each quadratic dimension for a total of $6k_BT/2$.

$$M\overline{\hbar\omega} = M\cdot\overline{\left(\frac{m}{2}\left(\frac{x}{t}\right)^2 + \frac{m}{2}\left(\frac{y}{t}\right)^2 + \frac{m}{2}\left(\frac{z}{t}\right)^2 + \frac{1}{2m}(p_x)^2 + \frac{1}{2m}(p_y)^2 + \frac{1}{2m}(p_z)^2\right)} = \frac{6\cdot k_B T}{2}$$

Since the average of x and the average of p_x is equal to zero (and for the other dimensions as well), the average of x^2 and p_x^2 is equal to the variance Δx^2 and Δp_x^2, which has already been calculated.

$$M\cdot\left(3\cdot\frac{m}{2}\cdot\frac{(\Delta x)^2}{(Kdt)^2} + 3\cdot\frac{(\Delta p_x)^2}{2m}\right) = M\cdot 6\cdot\frac{mc^2}{2K} = \frac{6\cdot k_B T}{2}$$

$$K = M\frac{mc^2}{k_B T}$$

With $K = M\cdot mc^2/k_B T$ (where M is a positive integer) and assuming that the particle is non-relativistic (mc^2/kT is much much greater than one), we are assured that the K>>1. This is nice since we already used this fact when exchanging the Gaussian as the limit of the binomial distribution.

Going ahead and plugging K into our equation for $\wp_K(\omega)d\omega$, we get $\wp_M(\omega)d\omega$

$$\wp_M(\omega)d\omega = \frac{\hbar\omega^3}{4\pi^2c^2}e^{\frac{-M\hbar\omega}{k_BT}}d\omega$$

We are only one more step away which is to sum over all states M from 1 to ∞, since all are possible.

$$\wp(\omega)d\omega = \sum_{M=1}^{\infty}\wp_M(\omega)d\omega = \frac{\hbar\omega^3 d\omega}{4\pi^2c^2}\sum_{M=1}^{\infty}e^{\frac{-M\hbar\omega}{k_BT}} = \frac{\hbar\omega^3}{4\pi^2c^2}\frac{1}{\left(e^{\hbar\omega/kT}-1\right)}d\omega$$

Appendix - Discrete model: Given our hypothesis that time and space are quantized, it is natural to model this process on a computer. The Matlab code below generates the Bernoulli process for each of the six dimensions at each time step and combines them together vis-á-vis the available energy. As shown in Figure (4) the resulting histogram is in very strong agreement with the known solution to black body radiation. Shown are the histogram of photons and the graph of $\wp(\omega)/\hbar\omega$ (the distribution of photons).

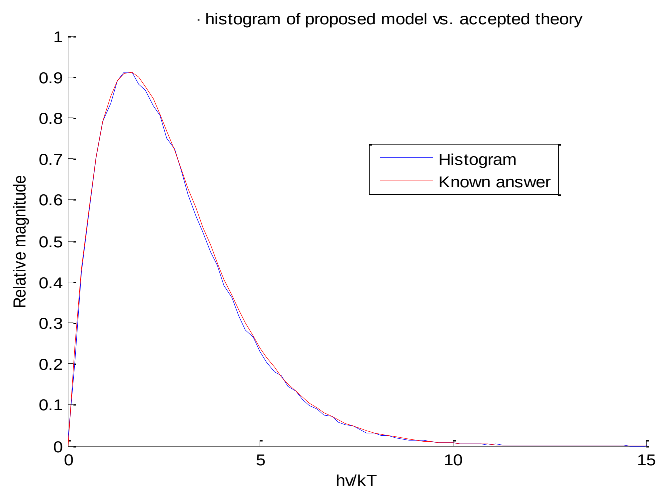

Figure 4 - Output of simulation vs. accepted theory

```
m=1;                        %Define unit mass assume speed of light is one, c=1
kT=m/1000;                  %Define Temperature << mass energy
dt=1/2/m;                   %Set time step where ħ=1, and c=1
dx=dt;                      %Set position step where speed of light is one
hv=kT*15*(.01:.01:1);       %Set energy independent variable's range of 15 units of temperature
N=150000;                   %Set number of random iterations

for M=1:10                  %Possible photon states, should be from M = 1:∞

    K=m*M/kT;               %For given photon state, set number of steps
    dp=m/K;                 %Set momentum step

    x=sum(dx*(2*floor(rand(K,N)-.5)+1));    %Generate P samples and sum over K steps for x
    y=sum(dx*(2*floor(rand(K,N)-.5)+1));    %Generate P samples and sum over K steps for y
    z=sum(dx*(2*floor(rand(K,N)-.5)+1));    %Generate P samples and sum over K steps for z
    px=sum(dp.*(2*floor(rand(K,N)-.5)+1));  %We use the Binomial to generate the Gaussian for px
    py=sum(dp.*(2*floor(rand(K,N)-.5)+1));  %We use the Binomial to generate the Gaussian for py
    pz=sum(dp.*(2*floor(rand(K,N)-.5)+1));  %We use the Binomial to generate the Gaussian for pz

    w=m/2*(x/(K*dt)).^2+m/2*(y/(K*dt)).^2+m/2*(z/(K*dt)).^2+px.^2/2/m+py.^2/2/m+pz.^2/2/m;
    %calculate excess energy

    h(:,M)=hist(w,hv)*2/(4*pi*(K*dx)^2)/(K*dt)/(4*pi);    %Calculate histogram of energy
    normalized for surface area, time between emissions, solid angle, and factor of two
end

H=sum(h');                  %Sum over energy states
P=hv.^2./(exp(hv/kT)-1);     %Calculate known intensity of photons
P(1)=0;
H=H/max(H);                 %Normalize for relative magnitude
P=P/max(P);                 %Normalize for relative magnitude
l=length(H);
rms=sqrt(sum((H-P).^2)/l);  %Compute RMS error of H with respect to I
figure
```

```
hold on
plot(hv,H)                    %Plot Histogram vs. frequency
plot(hv,P,'r')                %Plot Known theory vs. frequency
```

Appendix – Additional Questions: These advances still leave many questions outstanding, including (but not limited to):

1) Is the vacuum quantized in space and if so how does the location of a particle map to the grid?
2) What causes the radiation to be released at only certain time steps?
3) What is the cooling process when an *individual* particle emits black body radiation?
4) What is the true form of a particle and does it interact with other particles while it is stepping?
5) What happens if a particle is relativistic?

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